

Relation between electroabsorption in bulk semiconductors and in quantum wells: The quantum-confined Franz-Keldysh effect

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We evaluate the interband optical absorption of a semiconductor quantum well in the presence of a uniform electric field perpendicular to the layer and neglecting excitonic effects. We show that this formally becomes the Franz-Keldysh effect in the limit of an infinitely thick layer. When the potential drop across the layer is small compared to the confinement energy we obtain behavior qualitatively different from the bulk Franz-Keldysh effect and we explain this in terms of a quantum-confined Franz-Keldysh effect; with increasing field we demonstrate numerically for a GaAs-like semiconductor that we recover Franz-Keldysh-like behavior, once the originally "forbidden" quantum-well transitions become strong. Our discussion gives an alternative physical picture for the Franz-Keldysh effect, including a simple explanation of the Franz-Keldysh oscillations.

I. INTRODUCTION

Recent studies of electric field dependence of optical absorption (electroabsorption) in quantum-well structures have shown effects qualitatively different from those observed in bulk semiconductors.^{1,2} The quantum-well effects have been successfully explained as a quantum-confined Stark effect (QCSE).^{1,2} This effect is so different in character from the effects seen in bulk material (such as the Franz-Keldysh effect³⁻⁵) that the relation between the two has not been obvious. In this paper we demonstrate theoretically that, at least in the simplest approximation where excitonic effects are neglected, the two are fundamentally related. This gives an alternative picture for the Franz-Keldysh effect, and shows that, when excitons are neglected, the quantum-well effects can also be viewed as extreme quantization of the Franz-Keldysh effect.

Franz³ and Keldysh⁴ independently proposed the existence of an electric-field-dependent absorption "tail" in bulk semiconductors, commonly explained as photon-assisted tunneling of electrons from the valence to the conduction band. Later workers (see, for example, Dow and Redfield,⁶ and Merkulov and Perel⁷) noted that any full description of such an interband optical absorption should include the effects of the Coulomb interaction of the electron and hole. This results in exciton resonances and enhancement of the optical absorption above the optical band gap with no applied field. With applied field, the first consequence is a broadening of the exciton absorption line, together with some small shift of the exciton resonance to lower photon energies. At low fields, these effects can be viewed, respectively, as lifetime broadening of the exciton due to field ionization,⁸ and as a Stark shift of the ground ($1S$) state of the exciton, considered as a hydrogenic system, although this latter shift is relatively small ($\lesssim 10\%$ of the exciton binding energy)

before the resonance effectively ceases to exist.⁶ At high fields and at photon energies not too close to the band-gap energy, the simpler Franz-Keldysh effects are recovered in the limit.^{7,8} This theory⁶⁻⁸ provides a relatively complete description of electroabsorption, including excitonic effects, in a bulk semiconductor.

Investigations of electroabsorption near the optical band gap of quantum-well structures show qualitatively different electroabsorptive behavior for fields perpendicular to the layers.^{1,2,9} (The behavior for fields in the plane of the layers is qualitatively similar to that seen with bulk semiconductors.²) Specifically, in GaAs/ $\text{Al}_{1-x}\text{GaAs}$ quantum wells with layer thicknesses of the order of 100 Å, electric fields applied perpendicular to the layers result in large shifts in the optical absorption to lower photon energies, with the exciton resonances remaining well resolved. The shifts in exciton energy can be as much as 4 times the exciton binding energy,⁹ with little or no broadening so far resolved. These effects have been explained in terms of the Stark shift of a strongly confined hydrogenic system [QCSE (Refs. 1 and 2)]. The excitons are not field-ionized because, (1) the electrons and holes do not tunnel rapidly out of the wells, and (2) even when the electrons and holes are pulled to opposite sides of the layers, there is still a strong Coulomb attraction between them because the layers are so thin. Independent calculations by Brum and Bastard¹⁰ arrived at essentially similar conclusions. It is important in the calculations in the quantum well case that the confinement is strong, i.e., the exciton binding energy is small compared to the separation between the quantum-confined subbands. This means that the perturbation from the electron-hole Coulomb attraction on the electron and hole wave functions in the confinement direction can be neglected and the problem can be separated.^{1,2} Because it is difficult to treat the problem without such a separation, it is not easy to make the connection between the quantum-confined

problem and the bulk (unconfined) problem in the general case with excitonic effects included. However, as we will demonstrate below, a useful insight into the electroabsorption in both confined and unconfined cases can be obtained by solving for the quantum-well electroabsorption, neglecting the Coulomb interaction. Also, it is worth noting that the energy shifts in the QCSE are largely dominated by the shifts of the single-particle energies² (as would be obtained by neglecting the Coulomb interaction), so this approach may be relatively useful in describing the quantization of the energy spectrum, even if it does not completely describe the absorption.

In Sec. II of this paper, we will present the formal theory of electroabsorption in a thin slab, neglecting the Coulomb interaction, and will demonstrate that in the limit, as the slab becomes thick, we recover the Franz-Keldysh absorption. In Sec. III, we will present illustrative numerical results showing the transition between strongly quantized electroabsorption and the Franz-Keldysh effect. We will present our conclusions in Sec. IV.

II: ELECTROABSORPTION IN A SLAB

If we consider no interaction between electrons and holes, then the eigenstates for electrons and holes can be separately determined. We presume that the motion in the plane of the layers (i.e., perpendicular to the field) is still described by plane-wave propagation. For simplicity, we assume isotropic, nondegenerate, and parabolic bands, and will consider only one type of hole. For the motion perpendicular to the layers (z direction) we have the Schrödinger equations in the effective-mass approximation,

$$\left[-\frac{\hbar^2}{2m_i} \frac{d^2}{dz_i^2} \pm eFz_i \right] \phi_{in}(z_i) = E_{in} \phi_{in}(z_i), \quad (1)$$

where $i=e$ (electron) or h (hole), m_i is the particular effective mass, F is the electric field, n indexes the n th confined level with energy E_{in} , and $\phi_{in}(z_i)$ is the associated eigen-wave-function. The $+$ and $-$ signs refer to electrons and holes, respectively. We consider a slab of thickness L with infinite potential barriers for electrons and holes on either side, so we obtain the boundary conditions

$$\phi_{in} \left[\pm \frac{L}{2} \right] = 0, \quad (2)$$

where we choose the origin in the middle of the slab.

The solutions to Eq. (1) with boundary condition (2) are Airy functions²

$$\phi_{in}(z_i) = a_{in} \text{Ai}(Z_{in}) + b_{in} \text{Bi}(Z_{in}) \quad (3)$$

with

$$Z_{in} = - \left[\frac{2m_i}{(e\hbar F)^2} \right]^{1/3} (E_{in} \mp eFz_i).$$

Applying the boundary conditions gives the determinantal condition for solutions

$$\text{Ai}(Z_+) \text{Bi}(Z_-) - \text{Ai}(Z_-) \text{Bi}(Z_+) = 0 \quad (4)$$

(Z_+ and Z_- correspond to $z_i = +L/2$ and $-L/2$, respectively). Solving Eq. (4) gives the energies E_{in} . For each E_{in} we may deduce

$$\frac{b}{a} = - \frac{\text{Ai}(Z_+)}{\text{Bi}(Z_+)} = - \frac{\text{Ai}(Z_-)}{\text{Bi}(Z_-)} \quad (5)$$

to solve for the (unnormalized) wave functions.

Given these solutions E_{in} and ϕ_{in} , we may now evaluate the optical absorption or, equivalently, the imaginary part of the optical susceptibility, $\text{Im}\chi$, to obtain

$$\text{Im}\chi = \frac{C}{L} \sum_{\mathbf{k}_{||}, n, n'} \delta(E - E_g - \hbar^2 k_{||}^2 / 2m - E_{en} - E_{hn'}) I_{nn'}, \quad (6)$$

where

$$I_{nn'} = \frac{\left| \int_{-L/2}^{L/2} \phi_{en}(z) \phi_{hn'}(z) dz \right|^2}{\int_{-L/2}^{L/2} |\phi_{en}(z)|^2 dz \int_{-L/2}^{L/2} |\phi_{hn'}(z)|^2 dz} \quad (7)$$

and

$$C = \frac{2\pi\hbar^2 |p_{cv}|^2 e^2}{Am_0^2 E^2}.$$

In (6), E is the photon energy, E_g is the band-gap energy, and $\mathbf{k}_{||}$ is the electron-hole relative momentum in the plane of the layers, $1/m = 1/m_e + 1/m_h$. In arriving at Eq. (6), we make the usual assumptions of direct allowed transitions with negligible photon momentum, and interband matrix element p_{cv} independent of $\mathbf{k}$. A is the area of the slab (giving a volume $V = AL$), and the other symbols have their usual meanings. In what follows, we treat C as a constant, as the variations in E will be small compared to E .

One additional assumption in Eq. (6) is the neglect of nonlocal response, an important point for quantum-well structures. In general, the optical susceptibility χ is determined by the probability amplitude of finding the electron and hole at the same (center-of-mass) site $\mathbf{R}$. In layered systems with no translational invariance perpendicular to the surfaces,

$$\chi = \chi(\mathbf{R}, \mathbf{R}') = \chi(\mathbf{R}_\perp, \mathbf{R}'_\perp; \mathbf{R}_{||} - \mathbf{R}'_{||}) \neq \chi(\mathbf{R} - \mathbf{R}').$$

Effects associated with this nonlocal behavior shall not concern us here. For a photon wavelength larger than the layer thickness, we may define a "quantum-well-averaged" susceptibility

$$\frac{1}{V} \int d^3\mathbf{R} \int d^3\mathbf{R}' \chi(\mathbf{R}, \mathbf{R}')$$

which yields exactly expression (6), particularly the overlap integral (7).

For the case of a finite thickness of the slab, it will be useful to rewrite (6) by performing the summation over $\mathbf{k}_{||}$ to obtain

$$\text{Im}\chi = \frac{CAg_{2D}}{L} \sum_{n, n'} \Theta(E - E_g - E_{en} - E_{hn'}) I_{nn'} \quad (8)$$

where Θ is the Heaviside function, $\Theta(x > 0) = 1$, $\Theta(x < 0) = 0$, and $g_{2D} = m/2\pi\hbar^2$. Using (8), we may calculate the absorption spectrum for any finite slab thickness. We will return to this in Sec. III. However, to establish the relation with the infinite slab (i.e., the Franz-Keldysh effect) we will use Eq. (6), saving the sum over $k_{||}$ for later.

Now, using Eqs. (1)–(7), we will evaluate the limit of $\text{Im}\chi$ as $L \rightarrow \infty$. First we solve (4) in the limit of large L to deduce E_{in} as a function of n . Using the asymptotic expansions for Ai and Bi for large Z ,¹¹

$$\text{Ai}(Z) \simeq \frac{1}{2\sqrt{\pi}} \frac{1}{Z^{1/4}} \exp(-\frac{2}{3}Z^{3/2}) \text{ as } Z \rightarrow +\infty,$$

$$\text{Bi}(Z) \simeq \frac{1}{\sqrt{\pi}} \frac{1}{Z^{1/4}} \exp(\frac{2}{3}Z^{3/2}) \text{ as } Z \rightarrow +\infty,$$

$$\text{Ai}(Z) \simeq \frac{1}{\sqrt{\pi}} \frac{1}{|Z|^{1/4}} \sin(\frac{2}{3}|Z|^{3/2} + \pi/4) \text{ as } Z \rightarrow -\infty,$$

$$\text{Bi}(Z) \simeq \frac{1}{\sqrt{\pi}} \frac{1}{|Z|^{1/4}} \cos(\frac{2}{3}|Z|^{3/2} + \pi/4) \text{ as } Z \rightarrow -\infty,$$

we obtain from Eq. (4), as $L \rightarrow \infty$,

$$\sin \left[\frac{2}{3\hbar} (2m_e eF)^{1/2} \left[\frac{L}{2} + \frac{E_{in}}{eF} \right]^{3/2} + \frac{\pi}{4} \right] \simeq 0, \quad (9)$$

$$\lim_{L \rightarrow \infty} (\text{Im}\chi) = \sum_{k_{||}} \lim_{L \rightarrow \infty} \frac{C}{L} \int_{-eFL/2}^{\infty} dE_e \int_{-eFL/2}^{\infty} dE_h \frac{dn}{dE_e} \frac{dn'}{dE_h} \delta(E - E_g - \hbar^2 k_{||}^2 / 2m - E_e - E_h) I_{nn'}. \quad (12)$$

The lower limit in the integrals is deduced from (10). Indexing with energies E_e and E_h , rather than n and n' , $I_{nn'} \equiv I(E_e, E_h)$. Therefore, using the constancy of dn/dE_i ,

$$\lim_{L \rightarrow \infty} (\text{Im}\chi) = C \sum_{k_{||}} \frac{dn}{dE_e} \frac{dn'}{dE_h} \lim_{L \rightarrow \infty} \frac{1}{L} \int_{-eFL/2}^{E_{||} + eFL/2} dE_e I(E_e, E_{||} - E_e) \quad (13)$$

where $E_{||} = E - E_g - \hbar^2 k_{||}^2 / 2m$. The upper limit on the integral is deduced from the δ -function argument in (12).

To evaluate $I(E_e, E_{||} - E_e)$ for large L , we first neglect the Bi function as discussed above, so the (unnormalized) wave functions become

$$\phi_i(E_i, z_i) \simeq \text{Ai} \left[- \left[\frac{2m_i eF}{\hbar^2} \right]^{1/3} \left[\frac{E_i}{eF} \pm z_i \right] \right], \quad (14)$$

where the $+$ ($-$) sign refers to the electron (hole). The normalization integral becomes, in the limit of large L ,

$$\begin{aligned} I_i(E_i) &\equiv \int_{-L/2}^{L/2} |\phi_i(E_i, z_i)|^2 dz_i \\ &\simeq \int_{-L/2}^0 \text{Ai}^2 \left[\left[\frac{2m_i eF}{\hbar^2} \right]^{1/3} \left[z - \frac{E_i}{eF} \right] \right] dz, \end{aligned} \quad (15)$$

so that the energies E_{in} can be deduced from

$$n = \frac{2}{3\pi\hbar} (2m_i eF)^{1/2} \left[\frac{L}{2} + \frac{E_{in}}{eF} \right]^{3/2} + \frac{1}{4}, \quad (10)$$

and the density of states in energy becomes

$$\frac{dn}{dE_i} \simeq \frac{1}{\pi\hbar} \left[\frac{m_i L}{eF} \right]^{1/2} \quad (11)$$

as $L \rightarrow \infty$. (Henceforth we drop the index n , writing E_i instead of E_{in} .) Note that this density of states is constant in E_i . Incidentally, it is worth noting from (5) that the fraction of the Bi Airy function in the wave function becomes vanishingly small in the limit as $L \rightarrow \infty$; the wave function is essentially the Ai Airy function, which satisfies the boundary conditions (2) with a node of its oscillatory behavior at $\mp L/2$, and the exponentially decaying behavior giving effectively zero (in the limit of large L) at $\pm L/2$, where the upper (lower) signs refers to the electron (hole).

Using (11), we may now replace the summations over n and n' in (6) with integrals over the energies E_i ,

where we neglect the contribution from 0 to $+L/2$ as it decays exponentially (considering explicitly only the electron signs for simplicity), giving a vanishingly small contribution to the integral for large L . Substituting the asymptotic expansion for the Ai function of a negative argument, we may evaluate $I_i(E_i)$ in the limit of large L , because those regions where the asymptotic expansion is invalid make a negligibly small contribution to the integral. We obtain for large L

$$I_i(E_i) \simeq \frac{dn}{dE_i} \left[\frac{\hbar^2}{2m_i} \right]^{2/3} (eF)^{1/3}. \quad (16)$$

Now we must evaluate the overlap integral

$$\begin{aligned} I_{eh}(E_e, E_h) &\equiv \int_{-L/2}^{L/2} \phi_e(E_e, z) \phi_h(E_h, z) dz \\ &= \int_{-L/2}^{L/2} \text{Ai} \left[- \left[\frac{2m_e eF}{\hbar^2} \right]^{1/3} \left[\frac{E_e}{eF} - z \right] \right] \text{Ai} \left[- \left[\frac{2m_h eF}{\hbar^2} \right]^{1/3} \left[\frac{E_h}{eF} + z \right] \right] dz. \end{aligned} \quad (17)$$

Using the integral representation of the Airy function,¹⁰

$$\text{Ai}(Z) = \frac{1}{\pi} \int_0^\infty \cos \left[\frac{t^3}{3} + Zt \right] dt,$$

we have been able to evaluate the integral in (17) in the limit of large L to obtain

$$I_{eh}(E_e, E_h) \simeq \left[\frac{\hbar^2}{2M} \frac{1}{eF} \right]^{1/3} \times \text{Ai} \left[- \left[\frac{2meF}{\hbar} \right]^{1/3} \left[\frac{E_e + E_h}{eF} \right] \right], \quad (18)$$

where $M = m_e + m_h$. This direct mathematical proof is tedious and not very illuminating.

There is another mathematical proof of (18) that we will not give in detail here. If we consider the pair of equations (1) and transform them to relative and center-of-mass coordinates (as is normally done for the Franz-Keldysh case⁵), we then obtain two different solutions of the same problem. Subjecting both solutions to the same boundary conditions must give equivalent results. The linear combination of particular separate electron and hole solutions to (1) that corresponds to the solution in relative and center-of-mass coordinates in an infinite medium is essentially the integral (17), and the equivalence of solutions leads to the result (18).

One important feature of the result (18) is that it depends only on the sum $E_e + E_h$ and not independently on E_e and E_h . In the solution in relative and center-of-mass coordinates as $L \rightarrow \infty$,⁵ the center-of-mass motion is unperturbed by the field, and is zero anyway for states excited optically; only the total relative motion energy $E_e + E_h$ is relevant and thus $I_{eh}(E_e, E_h) = I_{eh}(E_e + E_h)$. In the solution with individual electron and hole coordinates, as performed in this paper, we first note an important property of the solution (14): changing the energy E_i only translates the solution in the z direction. Therefore, if in (17) we increase E_e and decrease E_h by the same amount, the result is simply to translate both Ai functions laterally in the same direction by the same amount. Hence there is no change in the overlap integral (17) when the limits are infinite. Consequently, again $I_{eh}(E_e, E_h)$ depends only on the sum $E_e + E_h$.

Our solution in z_e and z_h coordinates also gives insight into the reasons for the existence of the Franz-Keldysh "oscillations" on absorption (i.e., the "ripple" in absorption as a function of photon energy) above the band-gap energy. For photon energies below the band gap, the overlap of the electron and hole Airy functions (14) is such that the oscillatory tail of one function overlaps the exponential tail of the other; as the photon energy (and hence the sum $E_e + E_h$) is varied, there is no oscillatory behavior in the overlap. However, for photon energies above the band gap, $E_e + E_h$ is positive and there is a region of overlap of the oscillatory tails. Varying this overlap (by varying photon energy and hence $E_e + E_h$) results in the Franz-Keldysh oscillations of the total overlap integral and hence the optical absorption. The characteristic "period" of these oscillations (they are not strictly

periodic) corresponds to the energy change ΔE in $E_e + E_h$ required to move the Airy functions past one another by one "cycle" [e.g., moving from the i th peak of one function coinciding with the j th peak of the other to the $(i+1)$ th peak of one coinciding with the $(j+1)$ th of the other]. In each case, a period corresponds to a change in argument of order 1 so the characteristic energy is $\Delta E \sim E_0 \equiv (eF)^{2/3} (\hbar^2/2m)^{1/3}$. With increasing photon energy, the oscillations average out because the Airy functions are not strictly periodic.

Using Eqs. (16) and (18), we can now write Eq. (7) as

$$I(E_e, E_h) \equiv \frac{|I_{eh}(E_e, E_h)|^2}{I_e(E_e)I_h(E_h)} = \frac{1}{E_0^2 \frac{dn}{dE_e} \frac{dn'}{dE_h}} \left| \text{Ai} \left[- \frac{E_e + E_h}{E_0} \right] \right|^2. \quad (19)$$

Now substituting (19) into (13), we note that I is a function only of $E_{\parallel}$, which depends only on constants independent of E_e . Hence we obtain

$$\lim_{L \rightarrow \infty} (\text{Im}\chi) = C \sum_{k_{\parallel}} \frac{eF}{E_0^2} \left| \text{Ai} \left[- \frac{E_{\parallel}}{E_0} \right] \right|^2. \quad (20)$$

Transforming the sum to an integral we obtain

$$\lim_{L \rightarrow \infty} (\text{Im}\chi) = C A g_{2D} \frac{eF}{E_0} \int_{(E-E_g)/E_0}^{\infty} |\text{Ai}(x)|^2 dx. \quad (21)$$

This integral is evaluated readily using an identity derived directly from the Airy differential equation⁵ to obtain, finally,

$$\lim_{L \rightarrow \infty} (\text{Im}\chi) = C A g_{2D} \frac{eF}{E_0} \left[\frac{E - E_g}{E_0} \left| \text{Ai} \left[- \frac{E - E_g}{E_0} \right] \right|^2 + \left| \text{Ai}' \left[- \frac{E - E_g}{E_0} \right] \right|^2 \right], \quad (22)$$

which is identical with the result for the Franz-Keldysh effect.⁵ Hence we have proved explicitly that the absorption between confined electron and hole states in a (perpendicular) field tends to the Franz-Keldysh effect as the confined layer becomes thick. The perpendicular-field electroabsorption in the thin slab, in this approximation where electron-hole interaction is neglected, can therefore be described as the quantum-confined Franz-Keldysh effect.

III. NUMERICAL RESULTS

The formal proof in the preceding section that the electroabsorption of a slab of finite thickness tends towards the Franz-Keldysh effect, in the limit as the slab becomes very thick, does not indicate how this takes place or how thick a slab is required for Franz-Keldysh-like behavior, to some degree of approximation. To provide a qualitative answer and to give a further illustration, we have calculated the absorption directly from Eq. (8) [or from Eq. (22), in the case of an infinitely thick slab]. We choose a nontrivial example, considering a simple but realistic

semiconductor with single (nondegenerate) valence and conduction bands with infinite confining potential barriers and with parameters corresponding to heavy holes and electrons in GaAs (i.e., $m_h = 0.34m_0$, $m_e = 0.0665m_0$, where m_0 is the free-electron mass). This choice gives some practical feel for the magnitude of the effects, and the choice of masses that are not equal avoids accidental coincidences in the transition energies, preserving a unique energy for each transition. The results presented here are all calculated for 10^5 V/cm. All absorptions are per unit thickness, and neglect excitonic effects.

The calculations are performed by first solving (4) for the set of energies E_i up to some limit $E_{\max}$ for both electrons and holes, and then deducing the fractions of Bi Airy function from (5). Then, the wave functions for each state are normalized and all the overlap integrals $I_{nn'}$ are evaluated by direct numerical integration. Finally, the transitions are ordered in energy and the absorption is calculated from (8) up to a photon energy $E_{\max}$, above the zero-field bulk band gap. In performing the calculations, it is natural to use dimensionless units z/L for length, E/E_{1i} for energy,² where E_{1i} is the first confined state in a given band [$E_{1i} = (\hbar^2/2m_i)(\pi/L)^2$], and F/F_{1i} for field, where eF_{1i} corresponds to a potential drop of E_{1i} over the well width L , i.e., $F_{1i} = \pi^2\hbar^2/(2m_i eL^3)$. The solutions presented for, say, a 100-Å slab at 10^5 V/cm are therefore equivalent, for example, to those of a 1000-Å slab at 100 V/cm, with the energy units reduced by a factor of 100.

In the Franz-Keldysh effect, the phenomenon that received most attention is the tail induced in the optical absorption below the band gap (this was the only aspect considered in any detail by these original authors^{3,4}). For the comparison with the quantum-well case, where the optical absorption even with field is often totally above the zero-field bulk band gap, the oscillatory structure above the zero-field bulk band gap is also important (i.e., the Franz-Keldysh oscillations), and we start by considering this region in detail.

In Fig. 1 we show the absorption at zero field [Fig. 1(a)] and 10^5 V/cm [Fig. 1(b)] of a 100-Å slab, a 300-Å slab, and an infinitely thick slab. At zero field, the absorption shows the simple, quadratically spaced, steplike structure that results from only "allowed" transitions in a slab with infinitely high potential walls ($\Delta n = 0$) (i.e., subband n in the valence band to subband n in the conduction band). With field, however, we now observe more transitions, which are, of course, "forbidden" with $\Delta n \neq 0$. The 100-Å spectrum shows only a few, and shows the feature observed experimentally in such thin wells,^{1,2,9} namely that the optical absorption shifts to lower photon energy while retaining a large and abrupt onset, a phenomenon very different in character from the Franz-Keldysh effect.

However, by 300 Å, the spectrum is much richer, and closely resembles the Franz-Keldysh spectrum, including the oscillations, over this energy range. In these and in other simulations, we find that the thin-slab absorption starts to resemble the Franz-Keldysh effect above the band gap once the forbidden transitions become strong, *in which case they are often stronger than the allowed transitions*. This point is explicitly illustrated in Figs. 2 and 3.

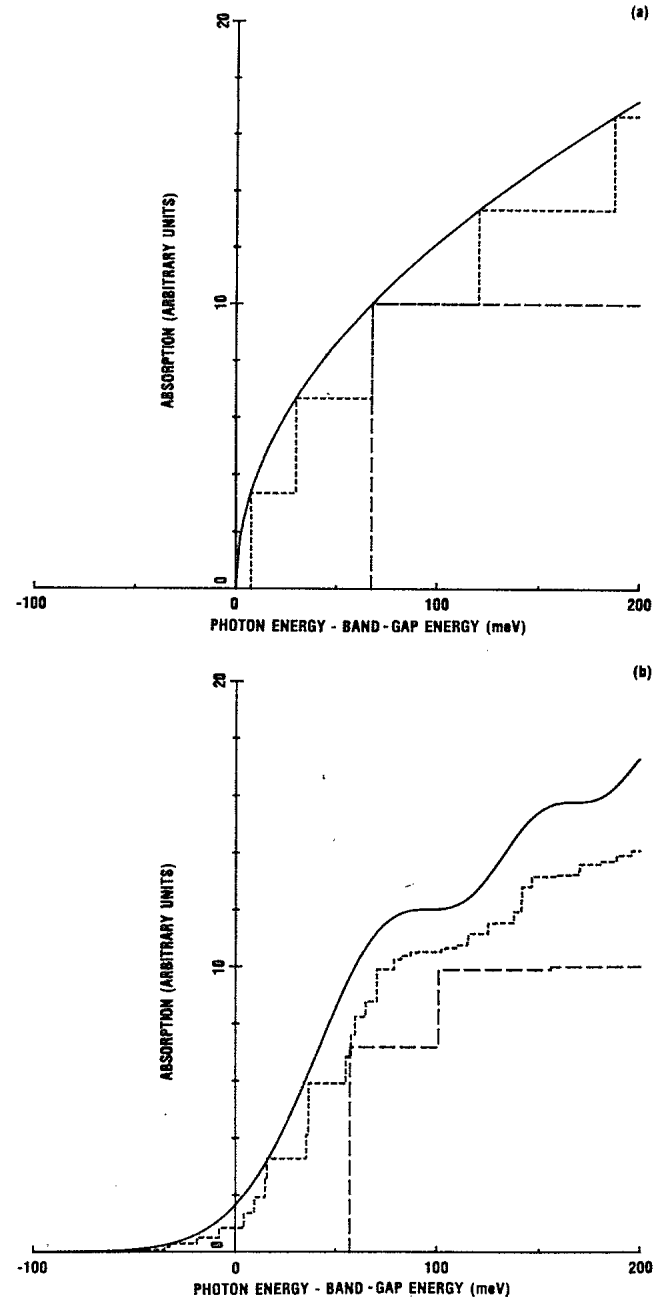


FIG. 1. Absorption of a 100-Å slab (long-dashed lines), a 300-Å slab (short-dashed lines), and an infinitely thick slab (solid lines), (a) at zero field, and (b) at 10^5 V/cm.

Figure 2 shows the calculated absorption of a 150-Å well at 10^5 V/cm, with the transitions labeled (n_v, n_c) ; n_v (n_c) is the number of the confined level in the valence (conduction) band numbered from the top (bottom). The corresponding energy levels and wave functions are illustrated in Fig. 3 for each of the levels involved in transitions in Fig. 2. The first Franz-Keldysh oscillation is just discernible in the 150-Å spectrum of Fig. 2, and some of the forbidden transitions [e.g., (1,2) and (2,1)] are stronger than the allowed transitions [e.g., (1,1) and (2,2)] in this energy region.

Figure 3 also illustrates several other points, especially

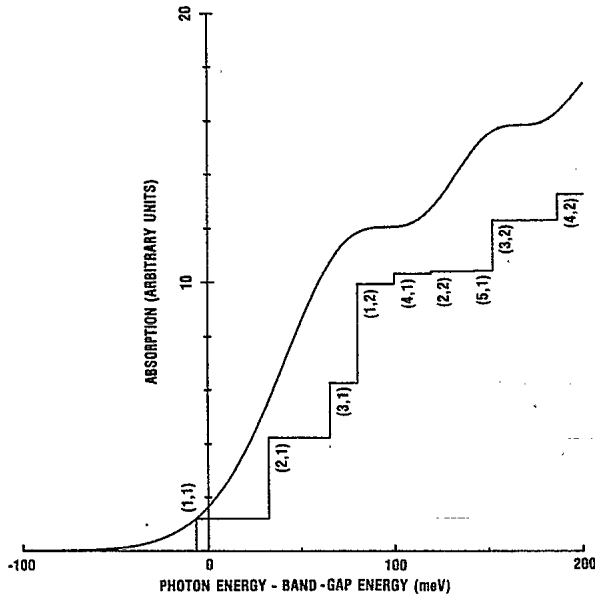


FIG. 2. Absorption of a 150-Å slab (stepped line) and an infinitely thick slab (smooth curve) at 10^5 V/cm. The steps are labeled (n_v, n_c) , where n_v (n_c) is the valence (conduction) subband number.

in the valence band, where the effect of field is a stronger perturbation. It is readily seen, for example, that the density of levels near the top of the valence band is much more nearly constant with field than without, as required in the limit [Eq. (11)]. Also, the first two wave functions for the hole are dominated by the Ai Airy function whose exponentially decaying tail effectively satisfies the boundary condition of zero wave function at the left wall. The higher levels have more Bi character in order to satisfy the boundary conditions, and start to approach

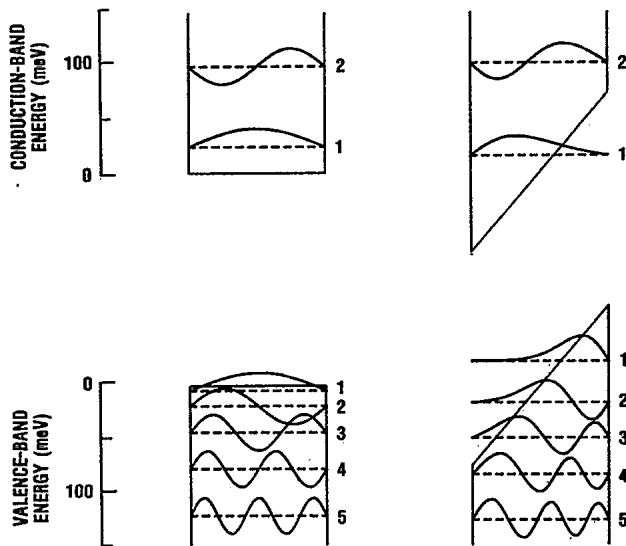


FIG. 3. Valence and conduction energy levels and normalized wave functions in a 150-Å slab at 0 and 10^5 V/cm, plotted together with the net confining potential (including the effect of field).

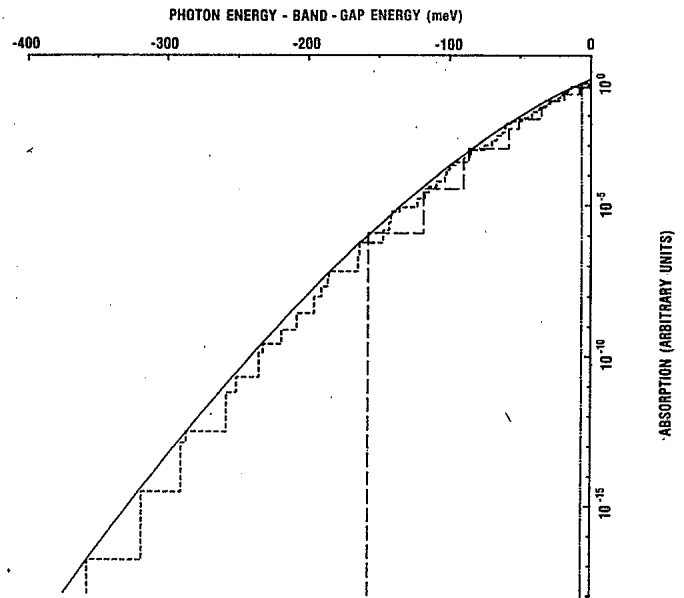


FIG. 4. Absorption on a logarithmic scale below the band gap for a 150-Å slab (dotted-dashed line), a 300-Å slab (long-dashed line), a 500-Å slab (short-dashed line), and an infinitely thick slab (solid line) at 10^5 V/cm.

sinusoidal behavior like the zero-field wave functions.

With sufficiently high photon energies, the higher, nearly sinusoidal wave functions start to dominate, with a consequent return towards the quadratically spaced step behavior rather than the approximately periodic Franz-Keldysh oscillations. The higher-lying states also shift very little with field, and the allowed transitions become relatively strong again.

Finally, in Fig. 4 we show the comparison at 10^5 V/cm between the thin-slab absorption and the Franz-Keldysh effect in the field-induced absorption "tail" below the band-gap energy, with absorption plotted on a logarithmic scale; this is the region normally associated with the Franz-Keldysh effect. The 150-Å slab on this plot does not closely resemble the Franz-Keldysh tail. However, the 300-Å slab, although showing clear steplike structure, is starting to mimic the Franz-Keldysh tail over more than 5 orders of magnitude, while the 500-Å slab essentially reproduces the tail over 16 orders of magnitude. The approach to Franz-Keldysh-like behavior is therefore very rapid with increasing slab width. The reason for this is easily seen from the scaling laws discussed above. For a given actual field, the dimensionless field scales as L^3 with the energy-level structure scaling as $1/L^2$. Thus, whereas the dimensionless field is 1.77 units and 9.04 units for the electron and hole, respectively, for the 100-Å well at 10^5 V/cm, it is 221 units and 1130 units, respectively, for the 500-Å well.

IV. CONCLUSIONS

We have calculated the optical absorption of a semiconductor slab or quantum well of arbitrary thickness in the presence of an electric field perpendicular to the slab,

neglecting excitonic effects. Because the walls of the well are presumed to be infinitely high, the problem is well defined and the electron and hole possess true eigenstates. Unlike some previous approaches to the Franz-Keldysh effect, the calculation of optical absorption then reduces to the evaluation of overlap integrals of eigenstates without any need to consider time-dependent wave functions or tunneling.

As we increase the thickness of the slab at a fixed field, we see a smooth transition from the strongly quantized behavior at small thickness that bears little obvious relation to the Franz-Keldysh effect, to Franz-Keldysh-like behavior over many orders of magnitude in absorption, with only small and closely spaced steplike behavior due to the confinement. We have also proved analytically that the Franz-Keldysh effect is recovered exactly in the limit of large thickness.

The transition towards Franz-Keldysh behavior is accompanied by the growth of forbidden transitions between the quantum subbands. The Franz-Keldysh oscillations can be discerned quite easily in the spectra once the forbidden transitions start to dominate. At high photon energies the quantum-well, quadratically spaced steps dominate again. Incidentally, in this picture, the Franz-Keldysh oscillations can be explained as being due to the variations in overlap integral of the valence and conduction wave functions as their relative displacement is altered. (Note that the wave function of a given particle is independent of particle energy in the infinite-slab problem, except for a lateral displacement.)

Finally, we note that the transition from the strongly quantized behavior to the Franz-Keldysh behavior takes place over a very small range of thickness at a given field. For example, in our calculations at 10^5 V/cm for our two-band GaAs-like semiconductor, at 100 Å the absorp-

tion bears no obvious relation to the Franz-Keldysh effect, whereas by 150 Å the Franz-Keldysh oscillations are discernible, and by 300 Å the Franz-Keldysh absorption tail is already reproduced over 5 orders of magnitude.

The calculations in this paper are not meant to model any particular experimental conditions and are necessarily unrealistic in two ways. First, they assume infinitely high barriers on either side of the semiconductor layers. This is not too drastic a restriction in practice when considering quantum wells, where tunneling between layers is weak for the lower subbands, although the behavior above the band gap may be modified by the finite barrier height. A more serious restriction, which is also inherent in Franz-Keldysh effect calculations, is the neglect of excitonic effects. This is not likely to be very important for the positions of the energy levels, the error being restricted to a term of the order of the exciton binding energy at zero field, although it is important for the magnitude of the absorption. However, it is known that the excitonic resonances in bulk semiconductors are strongly weakened by fields,^{6,8} whereas those in quantum wells can be retained to very high fields [the QCSE (Refs. 1 and 2)]. Hence, including excitonic effects should further enhance the contrast between the strongly quantum-confined electroabsorption and that in a thick slab. In this regard, it is interesting to note that for the representative field chosen here (10^5 V/cm) for our GaAs-like semiconductor, the transition range from strongly confined to nearly unconfined electroabsorptive behavior (i.e., 100 Å→300–500 Å) is exactly that over which the exciton confinement (and the associated additional exciton enhancement) disappears (the three-dimensional exciton diameter is ~300 Å in bulk GaAs). Hence, the actual contrast between quantum-confined electroabsorption and the unconfined case should be even stronger than that shown here.

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