Refractive Fabry–Perot Bistability with Linear Absorption: Theory of Operation and Cavity Optimization

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Abstract—We present a theory of optical bistability for a Fabry–Perot cavity containing a medium with nonlinear refraction but linear absorption with only plane-wave and slowly varying envelope approximations. An analytic expression for the critical intensity I_c for the onset of bistability is derived and used to compare cavity designs. It is shown that 1) the important material parameter for minimum I_c is the *ratio* n_2/α, 2) the limit to I_c is set by limitations on finesse due to inhomogeneities rather than the absorption itself, and 3) the cavity design which gives lowest I_c for a given finesse is that for which the mirror transmissivity equals the absorption per pass; at high finesse this design leads to a total peak cavity transmission of $\frac{1}{4}$ when mirror reflectivities are equal.

I. Introduction

In experimental observations of optical bistability in intrinsic nonlinear Fabry–Perot interferometers [1]-[7], the effect of nonlinear refraction [1]-[7] usually dominates over nonlinear absorption [7] in giving rise to the bistability itself. This was realized in the first observations of optical bistability by Gibbs *et al.* [1] who proposed simple theoretical descriptions for each extreme case (i.e., absorption only and refraction only) applicable to an atomic system as the nonlinear medium. A theory of optical bistability involving only nonlinear refraction was also proposed by Marburger and Felber [8]. Following the theoretical work of Bonifacio and Lugiato [9], much theoretical activity has centered on absorptive and refractive (dispersive) bistability with a two-level system as the nonlinear medium [10], although it is difficult to generalize this theory to the more complicated quantum mechanical systems which may be encountered in practice where only phenomenological parameters describing the linear and nonlinear behavior of the system may be available [11]. One such system is the semiconductor InSb where nonlinear refraction is observed in the presence of significant but substantially linear absorption [12] through bandgap-resonant saturation [13]. This is not a special case, but is an example of a general limiting condition to be expected for many different types of saturable systems when not driven too close to resonance. For example, even in a simple two-level saturable system, linear absorption is always present off-resonance (falling off as $\sim 1/\Delta\omega^2$ where $\Delta\omega$ is the distance from line center) which, by partially saturating the system, affects the refractive index at the off-resonant frequency, giving a first-order nonlinear refractive index falling off as $\sim 1/\Delta\omega^3$; the resultant change in absorption is, however, concentrated more at frequencies near to the line center (the first-order nonlinear absorption falls off as $\sim 1/\Delta\omega^4$) and it is easily shown rigorously that the limiting off-resonant condition is nonlinear refraction with linear ab-

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sorption. Furthermore, since comparatively large changes in absorption are required to give rise to absorptive bistability, while only very small changes in refractive index are needed for switching because of the interferometric nature of Fabry-Perot cavities, even quite large relative changes in absorption will have relatively little effect on a bistable Fabry-Perot cavity operating primarily by nonlinear refraction. These arguments suggest an analysis of nonlinear Fabry-Perot action for a medium with linear absorption and nonlinear refraction, and this analysis is the first purpose of this paper.

The need for including linear absorption becomes particularly apparent, however, in any attempt to optimize cavity design to obtain, say, a low switching intensity for a given material. Taking the nonabsorptive plane-wave theory of refractive bistability [8] to its logical extreme leads to the absurd conclusion that bistability can be observed at arbitrarily low intensities simply by increasing the length of the nonlinear material in the cavity *ad infinitum*. This cannot be valid in practice because absorption (which is unavoidable in *any* system relying on a saturation effect) must eventually quench out the multiple beam interference which is essential to the Fabry-Perot action and without which, in turn, there can be no bistability in this type of system.

In this paper the equations describing nonlinear Fabry-Perot action for uniform plane waves for an active material with first-order nonlinear refraction and linear absorption are formally derived in Section II. The analysis and results are similar to those of Marburger and Felber [8], giving two equations for the Fabry-Perot transmission T which, when solved simultaneously, give T in terms of the incident intensity I_0: the absorption is, however, fully included in these equations and no "mean-field" approximation [8], [14] is made; we also allow for unequal cavity mirror reflectivities. To give a criterion to compare different cavity designs, we derive an explicit expression for the critical intensity for the onset of bistability I_c in Section III, and define "figures of merit" both for the material (β) and the cavity design (μ), larger "figures of merit" giving proportionately lower I_c. In Section IV, the optimization of equal-reflectivity cavities is discussed in detail. Even with equal reflectivities, μ is a function of two parameters (e.g., mirror reflectivity and material absorption) and there is no single optimum design; neither is there a unique way of doing the optimization. Because, in practice, cavity finesse is limited by imperfections, we argue for an optimization giving minimum I_c for a given finesse $\mathcal{F}$ and present analytic and numerical results for the resulting cavity design and performance. In Section V, the important relations are given in their simplified forms for high finesse and the validity of the high-finesse approximation is discussed. The conclusions for cavity design and material selection are summarized in Section VI and an illustrative calculation of I_c is given for the case of InSb.

II. Nonlinear Fabry-Perot Action with Linear Absorption and Nonlinear Refraction

This analysis is similar to that of Marburger and Felber [8], except for the addition of linear absorption, and so we outline it only briefly. The general conclusion is that linear absorption makes little difference in the form of the equations which describe the nonlinear Fabry-Perot transmission, except that the ranges of transmission and reflection are appropriately reduced and absorption must be properly accounted for in the definition of finesse.

Using the normal wave equation for nonlinear optics for a nonmagnetic insulating medium, and considering only plane-wave solutions propagating in the + and - z directions in a plane-parallel cavity, we obtain in the steady state for the electric field $\mathcal{E}$ (where the $e^{i\omega t}$ time dependence has been removed)

$$\frac{\partial^2 \mathcal{E}}{\partial z^2} + k^2 \mathcal{E} = \left(ik\alpha - \frac{4\pi\omega^2}{c^2}\eta\,|\mathcal{E}|^2\right)\mathcal{E} \tag{1}$$

where α is the intensity absorption coefficient, k is the propagation constant, ω is the angular frequency, and c is the velocity of light. The nonlinear polarization (with the $e^{i\omega t}$ variation again removed) is $\mathcal{P} = \eta|\mathcal{E}|^2\mathcal{E}$ where we assume an isotropic medium and η is a real constant describing nonlinear refraction.

Defining real forward and backward amplitude ($\mathcal{E}_F$ and $\mathcal{E}_B$) and phase (ϕ_F and ϕ_B) envelope functions through

$$\mathcal{E} = \mathcal{E}_F e^{i\phi_F} e^{-ikz} + \mathcal{E}_B e^{i\phi_B} e^{ikz}$$

and taking the slowly varying envelope approximation and averaging over some spatial periods leads, on equating real and imaginary parts, to the four equations

$$\frac{\partial \phi_F}{\partial z} = \frac{-2\pi\omega\eta}{n_0 c}\left[\mathcal{E}_F^2 + 2\mathcal{E}_B^2\right] \tag{2}$$

$$\frac{\partial \phi_B}{\partial z} = \frac{2\pi\omega\eta}{n_0 c}\left[\mathcal{E}_B^2 + 2\mathcal{E}_F^2\right] \tag{3}$$

$$\frac{\partial \mathcal{E}_F}{\partial z} = \frac{-\alpha}{2}\mathcal{E}_F \tag{4}$$

$$\frac{\partial \mathcal{E}_B}{\partial z} = \frac{\alpha}{2}\mathcal{E}_B. \tag{5}$$

We solve these equations taking the usual boundary conditions for a Fabry-Perot resonator. Solving from (2) and (3) for the roundtrip nonlinear phase change $\phi_B - \phi_F$ enables an effective mean internal intensity I_{eff} to be defined through

$$\phi_B - \phi_F = 2\gamma I_{\text{eff}} = \frac{6\pi\omega\eta}{n_0 c}\int_0^D \left[\mathcal{E}_B^2(z) + \mathcal{E}_F^2(z)\right] dz \tag{6}$$

where D is the length of the cavity and $\gamma = 24\pi^2\omega\eta D/n_0^2 c^2$.

Defining new parameters $A = 1 - e^{-\alpha D}$ (A is the intensity absorption per pass), R_F (R_B) is the intensity reflectivity of the front (back) mirror, $R_\alpha = (1 - A)\sqrt{R_F R_B}$ (R_α is the effective mean reflectivity), $R_{B\alpha} = (1 - A) R_B$ ($R_{B\alpha}$ is the effective reflectivity of the back mirror), $F = 4R_\alpha/(1 - R_\alpha)^2$, and solving (4) and (5), now gives the total Fabry-Perot fractional (intensity) transmission T

$$T = \frac{(1 - R_B)(1 - R_F)(1 - A)}{(1 - R_\alpha)^2} \cdot \frac{1}{1 + F\sin^2(\gamma I_{\text{eff}} - \delta)} \tag{7}$$

and fractional (intensity) reflection S

$$S = \frac{(R_F + R_B(1-A)^2 - 2R_\alpha)}{(1-R_\alpha)^2 + F\sin^2(\gamma I_{eff} - \delta)} \cdot \frac{1}{1 + F\sin^2(\gamma I_{eff} - \delta)} \tag{8}$$

where I_{eff} can now be explicitly written as

$$I_{eff} = I_0 \frac{A}{\alpha D} \cdot \frac{(1-R_F)(1+R_B)}{(1-R_\alpha)^2} \cdot \frac{1}{1 + F\sin^2(\gamma I_{eff} - \delta)} \tag{9}$$

(I_0 is the incident intensity).

Combining (7) and (9) gives the second equation, parametric in I_{eff}, for the Fabry-Perot transmission T

$$T = \frac{\alpha D}{A} \cdot \frac{(1-R_B)(1-A)}{(1+R_{B\alpha})} \cdot \frac{I_{eff}}{I_0}. \tag{10}$$

Equations (7) and (10), solved simultaneously to eliminate I_{eff}, describe the nonlinear Fabry-Perot action with linear absorption in a fashion analogous to Marburger and Felber's [8] equations (31) and (24), and the behavior of the nonlinear cavity can be similarly visualized by graphic solution of (7) and (10). Now, in calculating the finesse $\mathcal{F} = (\pi/2)\sqrt{F} = \pi\sqrt{R_\alpha}/(1 - R_\alpha)$, it is the effective mean-mirror reflectivity, including the effects of bulk absorption, which must be used.

III. Critical Intensity and Detuning for the Onset of Bistability

The two relations between T and γI_{eff}, (7) and (10), represent the usual periodic Airy function and a straight line through the origin, respectively (see Fig. 1). The graphic criterion which determines whether a bistable region exists at a given incident intensity is whether the straight line makes a single or a multiple intersection with the periodic function, only multiple intersections giving bistability [8]. The condition for the existence of bistability, given suitable cavity mistuning, is that the slope of the straight line be less than the maximum slope of the periodic function. Since a shallower slope on the straight line corresponds to a greater incident intensity, this gives a condition to describe the minimum incident intensity below which bistability cannot be observed, regardless of cavity mistuning, i.e., the critical intensity for the onset of bistability I_c; the critical mistuning δ_c is that for which the critical straight line meets the curve at the point of equal gradient (see Fig. 1). To obtain actual bistability, it is necessary only to increase I_0 slightly above I_c and δ slightly above δ_c.

We can now solve formally for I_c and δ_c. The maximum gradient of the periodic function is, from differentiation of (7),

$$\left(\frac{\partial T}{\partial I_{eff}}\right)_{max} = \frac{16\gamma}{\sqrt{2}} \cdot \frac{(1-R_B)(1-R_F)(1-A)}{(1-R_\alpha)^2} \cdot \frac{H(F)}{[G(F)]^2} \tag{11}$$

where

$$H(F) = [(F+2)\sqrt{(F+2)^2 + 8F^2} - (F+2)^2 - 2F^2]^{1/2}$$

and

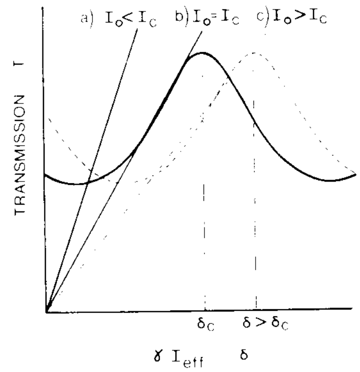

Fig. 1. Diagram illustrating that (a) for $I < I_c$ the straight line only ever makes one intersection with the curve; (b) the critical condition $I_0 = I_c$, $\delta = \delta_c$; (c) bistability (multiple intersection of line and curve) for $I_0 > I_c$, $\delta > \delta_c$.

$$G(F) = 3(F+2) - \sqrt{(F+2)^2 + 8F^2}.$$

This maximum gradient occurs when

$$\sin^2(\gamma I_{eff} - \delta) = \frac{1}{4F}(3F + 2 - \sqrt{(F+2)^2 + 8F^2}). \tag{12}$$

Equating $(\partial T/\partial I_{eff})_{max}$ to the gradient of the "straight line" [from (10)] gives the expression for I_c:

$$I_c = \frac{\sqrt{2}}{16\pi\beta} \cdot \frac{(1-R_\alpha)^2}{A(1+R_{B\alpha})(1-R_F)} \cdot \frac{[G(F)]^2}{H(F)} \tag{13}$$

where $\beta = \gamma/\pi\alpha D$.

Substituting the value at maximum gradient of $\sin^2(\gamma I_{eff} - \delta)$ from (12) and I_c for I_0 in (9) gives the critical effective internal intensity $I_{eff\,c}$:

$$\gamma I_{eff\,c} = \frac{\sqrt{2}}{4} \cdot \frac{G(F)}{H(F)} \tag{14}$$

and, hence, from (12) we deduce the critical value δ_c of the cavity mistuning δ:

$$\delta_c = \frac{\sqrt{2}}{4} \cdot \frac{G(F)}{H(F)} - \sin^{-1}\left[-\left\{\frac{1}{4F}(3F + 2 - \sqrt{(F+2)^2 + 8F^2}\right\}^{1/2}\right]. \tag{15}$$

The sine itself is always negative at the point of maximum positive slope on the periodic function, and thus the negative square root is taken in the $\sin^{-1}$ term. δ_c depends *only* on the parameter F, or, alternatively, on the cavity finesse $\mathcal{F} = \pi\sqrt{R_\alpha}/(1 - R_\alpha) = \pi/2\sqrt{F}$, and this dependence is shown in Fig. 2. ($\delta_c > \pi$ implies that bistability cannot be obtained in the first nonlinear order of the cavity.)

In expression (13) for I_c, all the relevant material properties are contained in one parameter β which can conveniently be rewritten as $\beta = 3n_2/\lambda\alpha$ where n_2 is defined through $n = n_0 + n_2 I$. Within the limitations of this plane-wave model, β is the single "figure of merit" for the material, higher β giving lower I_c (for given A, R_A, and R_B). β can be interpreted as the limiting roundtrip nonlinear phase shift (in wavelengths) per unit intensity as the absorbing nonlinear material is made

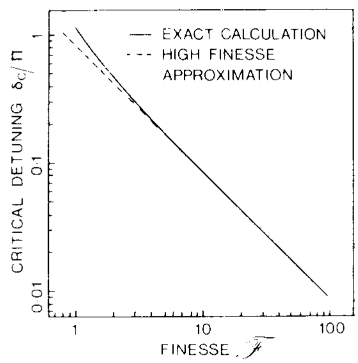

Fig. 2. Critical detuning of the cavity for the onset of bistability as a function of the cavity finesse.

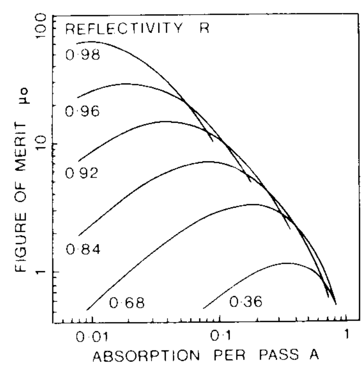

Fig. 3. Cavity figure of merit μ_0 (equal mirror reflectivities) as a function of mirror reflectivity R and absorption per pass A.

arbitrarily long (intensity is defined as the forward intensity just inside the front mirror).

All the other terms in (13) can be grouped conveniently into a cavity figure of merit μ which is a function of the three independent cavity parameters R_F, R_B, and A. Of course, any three independent functions of R_F, R_B, and A can also be used as cavity parameters. We choose, for the moment, R_α (or F), $R_{B\alpha}$, and A. This also enables us to split μ into two components, one of which is a function of only two parameters; this will make the comparison of cavity designs more straightforward. Thus, we have

$$I_c = \frac{1}{\beta} \cdot \frac{1}{\mu} \tag{16}$$

where

$$\mu(R_\alpha, R_{B\alpha}, A) = \rho(R_\alpha, R_{B\alpha}, A)\,\mu_0(R_\alpha, A) \tag{17}$$

$$\mu_0(R_\alpha, A) = \frac{16\pi}{\sqrt{2}} \cdot \frac{A(1+R_\alpha)(1-R_\alpha/(1-A))}{(1-R_\alpha)^2} \cdot \frac{H(F)}{[G(F)]^2} \tag{18}$$

$$\rho(R_\alpha, R_{B\alpha}, A) = \frac{(1+R_{B\alpha})(1-R_\alpha^2/R_{B\alpha}(1-A))}{(1+R_\alpha)(1-R_\alpha/(1-A))}. \tag{19}$$

In this description, μ_0 is the cavity figure of merit for equal mirror reflectivities, and the factor ρ describes how the overall cavity figure of merit μ is influenced by altering the relative reflectivities of the two mirrors for a given R_α and A (for $R_B = R_F$, $\rho = 1$).

The maximum possible value of ρ occurs (for given R_α and A) when $R_{B\alpha}$ is a maximum. The largest value of $R_{B\alpha}$ is when the back mirror reflectivity (R_B) is 100 percent, and then $R_{B\alpha} = 1 - A$. Therefore,

$$\rho_{max} = 2 - \frac{A(1-R_\alpha/(1-A))}{1+R_\alpha}. \tag{20}$$

In practice, the second term on the right-hand side of (20), although always positive (so that $\rho_{max} < 2$), is usually small (for $R_\alpha \gtrsim 0.5$, $\rho_{max} \gtrsim 1.8$ for all A). Therefore, the effect of altering the relative reflectivities of the mirrors can give a reduction in I_c for any given R_α and A of up to a factor of $\lesssim 2$. This is done of course at the expense of cavity transmission, which, in the extreme case (R_B = 100 percent), is zero. Note that this mode of operation (R_B = 100 percent) is only possible in the presence of absorption; otherwise the cavity reflectivity is always 100 percent [see (8)].

This enables us, in practice, to remove the factor ρ from further consideration in determining μ and removes one variable (R_B) from any optimization analysis; we now study the function μ_0, the figure of merit for cavities with equal front and back mirror reflectivities.

IV. Optimization of Cavity Figure of Merit μ_0

The general behavior of μ_0 as a function of the (equal) mirror reflectivities R and the absorption per pass A is shown in Fig. 3. The first property of μ_0 to note is that there is no single optimum cavity: by going to higher R we can always choose some A to obtain a higher μ_0; similarly, by going to lower A we can always choose some R to obtain a higher μ_0. These two optimization methods are, however, arbitrary and lead to different cavity designs. Also, while A and R are the final parameters used for manufacturing the cavity, they do not convey any obvious information about the behavior of the cavity. It is more meaningful to choose the two parameters T_{max}, the maximum cavity transmission ($T_{max} = (1-R)^2(1-A)/(1-R(1-A))^2$) and $\mathcal{F}$, the cavity finesse; these two parameters are mathematically independent functions of A and R and provide an adequate mathematical basis for describing cavity performances; furthermore, they are easily related to the observable behavior of the cavity, being a simple function of the contrast between maximum T_{max} and T_{min} transmission through

$$\frac{T_{max}}{T_{min}} = 1 + \frac{2\mathcal{F}^2}{\pi}. \tag{21}$$

The inversion formulas for A and R in terms of T_{max} and $\mathcal{F}$ are

$$R = 1 + \frac{\pi^2 T_{max}}{2\mathcal{F}^2} - \frac{\pi\sqrt{T_{max}}}{\mathcal{F}}\left(1 + \frac{\pi^2 T_{max}}{4\mathcal{F}^2}\right)^{1/2} \tag{22}$$

$$A = 1 - \frac{1}{R}\left[1 + \frac{\pi^2}{2\mathcal{F}^2} - \frac{\pi}{\mathcal{F}}\left(1 + \frac{\pi^2}{4\mathcal{F}^2}\right)^{1/2}\right]. \tag{23}$$

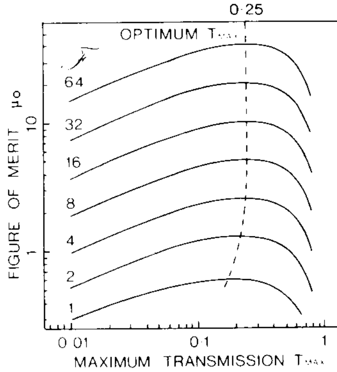

Fig. 4. Cavity figure of merit μ_0 (equal mirror reflectivities) as a function of peak (fractional) transmission T_{max} and cavity finesse. The dashed line shows the optimum value of T_{max} for each finesse.

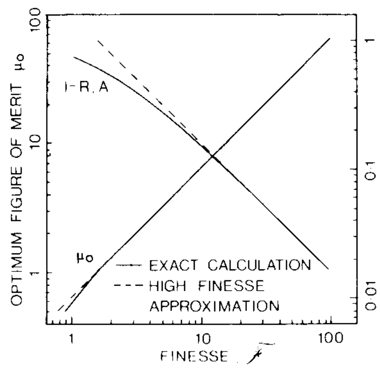

Fig. 5. Optimum cavity figure of merit μ_0 (equal reflectivities) for a given finesse as a function of that finesse, together with the corresponding design values of mirror transmissivity $(1 - R)$ and absorption per pass A, these two quantities, $(1 - R)$ and A, being exactly equal for this optimization.

The behavior of μ_0 as a function of T_{max} and $\mathcal{F}$ is shown in Fig. 4. Now we see that, if the optimum T_{max} is chosen for each finesse $\mathcal{F}$, then the figure of merit μ_0 increases monotonically with increasing $\mathcal{F}$. This tells us that we may obtain as high a μ_0 as desired, provided we can engineer a cavity with a sufficiently high finesse $\mathcal{F}$. In practice, the finesse obtainable from a cavity is limited by inhomogeneities in the cavity mirrors and medium. Note, however, that the fundamental limit on $\mathcal{F}$ is *not* imposed by the fact that we always have finite absorption in the cavity. Our calculations include the effect that, in order to obtain higher $\mathcal{F}$ and μ_0, we must use ever thinner nonlinear mediums (i.e., less absorption per pass). The limit on μ_0 and, hence, the limit on the minimum intensity at which bistability can be obtained for a given material, is set only by the limits of cavity technology.

If we assume, then, that we are practically limited to a particular maximum finesse $\mathcal{F}$, then one obvious optimization procedure is to choose T_{max} to give the maximum figure of merit for a fixed finesse; this is then seen to be a more useful procedure than arbitrarily choosing R at fixed A or A at fixed R.

From differentiation of (18) with respect to A at constant R_α (and, hence, constant F and $\mathcal{F}$), it is readily shown that the optimum choice of R and A for a given finesse is

$$R_{opt} = 1 - A_{opt} = \sqrt{R_\alpha} = \left(1 + \left(\frac{\pi}{2\mathcal{F}}\right)^2\right)^{1/2} - \frac{\pi}{2\mathcal{F}}. \tag{24}$$

This gives the very simple, exact conclusion that, if the mirror transmissivity $1 - R$ is chosen equal to the absorption per pass A, then the resulting cavity has the maximum figure of merit μ_0 for its finesse $\mathcal{F}$.

From (24) we can calculate the optimum maximum transmission T_{max} for a given finesse $\mathcal{F}$, giving

$$T_{max\ opt} = \frac{\left(1 + \left(\frac{\pi}{2}\right)^2\right)^{1/2} - \frac{\pi}{2\mathcal{F}}}{2\left[\left(\frac{\pi}{2\mathcal{F}}\right)^2 + \left(1 - \frac{\pi}{2\mathcal{F}}\right)\left(1 + \left(1 + \frac{\pi}{2\mathcal{F}}\right)^2\right)^{1/2}\right]}. \tag{25}$$

This relation is plotted as the dashed line in Fig. 4.

Equations (24) and (18) can be used to give an exact analytic relation for the optimum μ_0. The optimum μ_0, A_{opt}, and $1 - R_{opt}$ are plotted in Fig. 5 as a function of the finesse $\mathcal{F}$.

Note from (25) that there is an optimum transmission. If the cavity is designed for higher transmission in particular, the fall-off in μ_0 is rapid, and there is also nothing to be gained by going to cavity designs with lower transmission than the optimum.

V. High Finesse Approximation

In the limit of high finesse (i.e., small A and $1 - R$) the relations describing the absorbing nonlinear refraction Fabry-Perot become very much simpler, while still retaining forms similar to the exact results. We retain terms only up to the lowest order in A or $1 - R$ or in both, as appropriate.

The finesse $\mathcal{F}$ is approximated by

$$\mathcal{F} \simeq \pi/(1 - R + A) \tag{26}$$

and

$$T_{max} \simeq [(1 - R)/(1 - R + A)]^2. \tag{27}$$

The cavity figure of merit μ_0 [see (18)] simplifies to

$$\mu_0 \simeq \frac{3\sqrt{3}}{2}\pi\frac{A(1 - R)}{(1 - R + A)^3} \simeq \frac{3\sqrt{3}}{2}\mathcal{F}\sqrt{T_{max}}(1 - \sqrt{T_{max}}) \tag{28}$$

and the critical detuning δ_c [see (15)] becomes

$$\delta_c \simeq \frac{\pi\sqrt{3}}{2\mathcal{F}} \simeq \frac{2.72}{\mathcal{F}} \tag{29}$$

(see Fig. 2) with $\gamma I_{eff\,c}$, the required critical nonlinear phase thickness change [see (14)], given by

$$\gamma I_{eff\,c} \simeq \frac{2}{3}\delta_c \simeq \frac{\pi}{\sqrt{3}\mathcal{F}} \simeq \frac{1.81}{\mathcal{F}}. \tag{30}$$

The conditions for the optimum figure of merit for a given finesse become, from (24)

$$1 - R_{opt} = A_{opt} \simeq \frac{\pi}{2\mathcal{F}} \simeq \frac{1.57}{\mathcal{F}} \tag{31}$$

(see Fig. 5) and, from (27)

$$T_{max\ opt} \simeq \tfrac{1}{4}. \tag{32}$$

The optimum figure of merit for a given finesse $\mu_{0\ opt}$ then becomes

$$\mu_{0\ \mathrm{opt}} \simeq \frac{3\sqrt{3}}{8}\ \mathcal{F} \simeq 0.65\,\mathcal{F} \tag{33}$$

(see Fig. 5).

As can be seen from Figs. 2, 4, and 5, the high finesse approximation is very good for $\mathcal{F} \gtrsim 10$. For relations (29), δ_c, and (33), $\mu_{0\ \mathrm{opt}}$, it remains a good description even down to $\mathcal{F} \sim 1$.

In the high finesse limit the behavior of the optimized equal reflectivity cavity $(1 - R = A)$ can be described particularly simply. For example, off-resonance, it reflects $1 - 3\pi^2/16\mathcal{F}^2$, absorbs $\pi^2/8\mathcal{F}^2$, and transmits $\pi^2/16\mathcal{F}^2$ of the incident light; at the critical detuning δ_c the values of reflectance, absorption, and transmission are $\frac{13}{16}$, $\frac{1}{8}$, and $\frac{1}{16}$, respectively (independent of $\mathcal{F}$), and on-resonance $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{1}{4}$, respectively.

VI. Discussion and Conclusions

Because the relations describing the cavity behavior [(7) and (10)] are analogous to those obtained for refractive bistability without absorption [8], the general behavior of the cavity will be similar, except that transmission and reflection are never unity and there is always some absorption inside the cavity. The definition of finesse, rather than depending only on the mean-mirror reflectivity R, must now include absorption, giving $\mathcal{F} = \pi\sqrt{R(1 - A)}/[1 - R(1 - A)]$ where A is the single pass absorption through the material. The critical mistuning δ_c for the onset of bistability is, however, only dependent on $\mathcal{F}$.

When (7) and (10) are solved for the critical intensity for the onset of bistability I_c, the important material parameter for a given choice of cavity parameters (i.e., reflectivities and absorption per pass) is $\beta = 3n_2/\lambda\alpha$; this is essentially the ratio of refractive nonlinearity to linear absorption and reflects the fact that, for example, with a material of, say, weaker absorption coefficient α, we can tolerate a correspondingly weaker nonlinearity n_2 because the material can be made correspondingly thicker inside the cavity, and the same switching intensity is retained.

In examining how the cavity can be designed to reduce the switching intensity, we conclude firstly that making the back cavity mirror more reflecting (while keeping the mean reflectivity constant) leads to $\lesssim 2$ times reduction in I_c at the expense of transmission. The cavity "figure of merit" for equal reflectivities depends only on R and A. However, there is no single optimum cavity design, except the limit $R \rightarrow 100$ percent $A \rightarrow 0$ when $I_c \rightarrow 0$. While high R and low A can be achieved, the more severe practical limitation is on finesse, higher finesse requiring higher optical perfection on materials and mirror surfaces. Optimizing to give minimum I_c for a given finesse leads to the simple *exact* design $1 - R = A$, i.e., mirror transmissivity is equal to absorption per pass. At high finesse this leads to $I_c \propto 1/\mathcal{F}$ and a maximum transmission of the Fabry-Perot of $\frac{1}{4}$.

As an illustrative calculation, InSb at 77 K at 1852 cm^{-1} has $n_2 = 3 \times 10^{-3}$ cm^2/W and $\alpha = 80$ cm^{-1} [6]. Allowing for the possibility that standing-wave effects may disappear due to diffusion reduces β by $\frac{2}{3}$ [8], giving $\beta = 0.14$ cm^2/W. For a finesse of ~30 with $R = 95$ percent and $A = 5$ percent (implying a sample thickness of 6 μm), I_c would be ~0.4 W/cm^2 (4 nW/μm^2).

In summary, we have been able, within the limitations of the plane-wave approximation, to solve for refractive nonlinear Fabry-Perot action in the presence of linear absorption, and to give criteria for optimizing the design of such systems for minimum switching intensity.

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