

# Quadratic electro-optic effect due to the quantum-confined Stark effect in quantum wells

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We have performed a semi-empirical calculation of the refractive index changes induced by a perpendicular electric field in quantum wells. This calculation is based on experimental electroabsorption data, together with absorption sum rules that have recently been developed. We find good agreement with published experimental results. Our results are important to the development of electro-optic and high-speed electroabsorption quantum well devices.

Recently, there has been considerable interest in the electric field dependence of optical properties in semiconductor quantum well (QW) structures. For electric fields perpendicular to the quantum well layers, the optical absorption near the band-gap energy can be shifted to lower photon energies without destroying the strong excitonic features; this effect, known as the quantum-confined Stark effect (QCSE),<sup>1</sup> is of practical interest for absorption modulators and switches (see, e.g., references summarized in Ref. 2) that operate at room temperature with low energies and at wavelengths compatible with laser diodes. It is also important to know the size of the changes in refractive index associated with this unusual electroabsorption. Large changes in index would make the QW's attractive for electro-optic devices such as directional couplers. Conversely, it is important that the changes in refractive index in high-speed absorption modulators not be so large that they introduce chirp in the transmitted beam. In this letter, we present calculations of the change of index with perpendicular field, based on experimental absorption data<sup>2,3</sup> and on absorption sum rules that have recently been derived.<sup>2</sup> We compare these with recent experimental measurements<sup>4,5</sup> and with a previous theoretical calculation.<sup>6</sup> We use these results to calculate the chirp of a high-speed modulator.<sup>7,8</sup>

The basic method of our calculation is to take measured changes in absorption with electric field and calculate using the Kramers-Kronig relations the associated changes in refractive index. The refractive index and absorption are related by<sup>9,10</sup>

$$n(\omega) - 1 = \frac{c}{\pi} P \int_0^\infty \frac{\alpha(\omega') d\omega'}{(\omega')^2 - \omega^2}. \quad (1)$$

In principle, use of Eq. (1) requires knowledge of all changes in absorption over the entire optical spectrum. In practice this is not possible. We can make several approximations which will permit a realistic calculation, however. First, we are only interested in index changes associated with the QCSE. All effects associated with the QCSE are of even order in the electric field. We therefore neglect the linear electro-optic effect<sup>4</sup> as this can be treated as a separate effect associated with transitions quite different from those treated here. Second, if the remaining changes in absorption occur in small and localized spectral regions, such as near the sub-

band absorption edges, then close to such a region the remaining refractive index change will be dominated by the local change in absorption. This is particularly true if the local oscillator strength is conserved, in which case the changes in refractive index fall off quite rapidly [ $\sim 1/(\Delta E)^2$ ] with change  $\Delta E$  in photon energy. (If area is not conserved then there are changes falling off only as  $\sim 1/\Delta E$ ). It has recently been observed empirically that in the electroabsorption spectra of quantum wells there is such a local conservation of area near the  $n = 1$  light-hole (lh) and heavy-hole (hh) excitons.<sup>2</sup> This has been theoretically justified by oscillator strength sum rules for excitonic and band-to-band absorption. Although the sum rules apply separately to each subband, there should also be overall conservation of area under the entire absorption spectrum. We therefore calculate the change in the refractive index near the band-gap energy due to the QCSE considering only the changes in absorption at and near the  $n = 1$  hh and lh exciton resonances, and, assuming that changes in absorption at other energies produce negligible change in index because they too are spectrally local. By substituting  $n(\omega) = n_0(\omega) + \Delta n(\omega)$  and  $\alpha(\omega) = \alpha_0(\omega) + \Delta\alpha(\omega)$  into Eq. (1) and assuming  $\Delta\alpha < > 0$  only in the energy range  $\omega_1 < \omega < \omega_2$  we find

$$\Delta n(\omega) = \frac{c}{\pi} P \int_{\omega_1}^{\omega_2} \frac{\Delta\alpha(\omega') d\omega'}{(\omega')^2 - \omega^2}. \quad (2)$$

The electroabsorption data were taken from previous measurements on a two 94-Å QW waveguide modulator<sup>3</sup> with polarization both parallel to the plane, in which both lh and hh excitons could be observed, and perpendicular to the plane, in which only the lh exciton appears. The latter polarization provides particularly clean spectra for analysis. Only two QW's were used in this structure so that the absorption length would be quite long. Since the refractive index changes are expected to be quite small, we will scale these data using the waveguide filling factor and layer thicknesses to arrive at refractive index estimates for multiple QW (MQW) material of equal well and barrier thicknesses.

Absorption spectra with and without field were subtracted to obtain the spectrum of the change of absorption,  $\Delta\alpha(\omega')$ , and numerical integration of Eq. (2) was per-

formed to obtain the resulting induced change in refractive index. The only adjustment made to the data was to constrain the conservation of area to be exact rather than within the experimental error of a few percent. This is justified by the sum rules described earlier. In general, the calculation should be valid as long as  $\Delta E$  is small compared to the energy separation between subbands, and on the low-energy side it will be valid for even larger  $\Delta E$  since the photon energy is far from the other transitions of the system.

Figure 1(a) shows the absorption spectra for  $\hat{e} \parallel$  plane at zero field (solid curve) and at  $6.5 \times 10^4$  V/cm (broken curve), together with the calculated change in refractive index (dot-dashed curve). Both hh and lh excitons appear in this polarization. The excitons shift to lower energy with applied field. The difference between the absorption spectra is quite small at the highest energy, justifying the integration

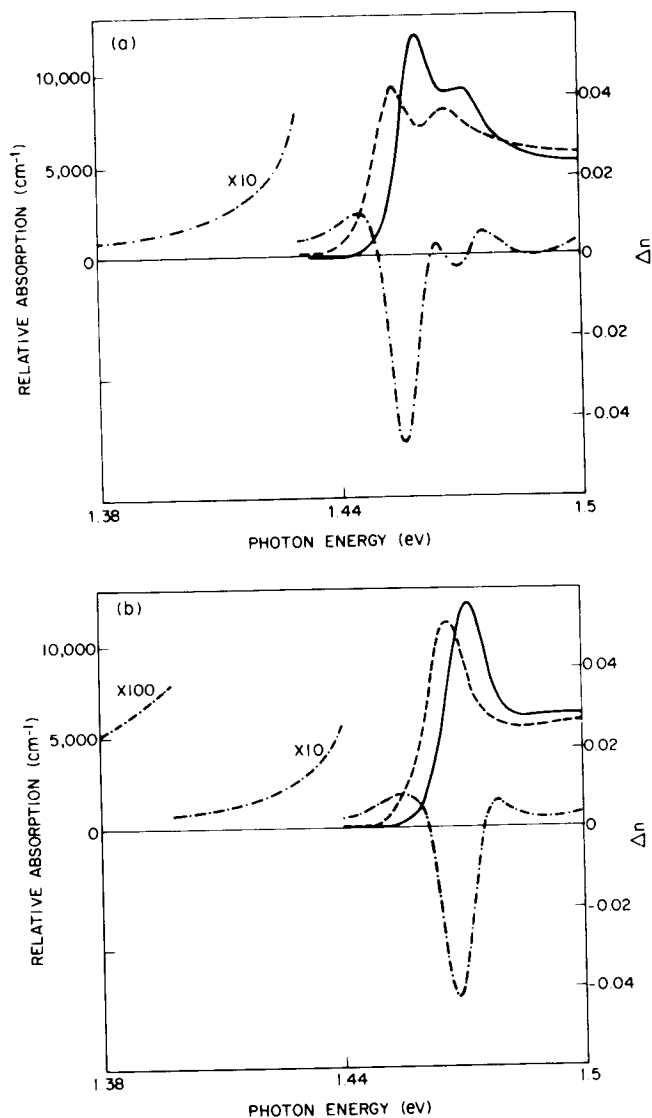


FIG. 1. Quantum well spectra for incident polarization (a) parallel to the layers and (b) perpendicular to the layers, respectively. The solid and broken curves are the absorption spectra taken at zero field and at  $6.5 \times 10^4$  V/cm, respectively. The dot-dashed curve is the calculated change in refractive index. The data was scaled to correspond to multiple QW material of equal well and barrier widths.

over limited energy range. Below the exciton  $\Delta n$  is positive, and decreases rapidly with energy offset from the exciton. For photon energies significantly below the exciton peaks, the decrease is quadratic in energy offset, as expected when oscillator strength is conserved. Near the hh exciton  $\Delta n$  is negative, corresponding to the zero crossing of the induced absorption, and reaches a very large amplitude of 0.04. There is also a small negative peak due to the lh exciton.

Recent electroreflectance measurements in the vicinity of the hh exciton<sup>5</sup> give a maximum refractive index change of approximately 0.06–0.07 for equal well and barrier widths at a field  $7 \times 10^4$  V/cm. In view of the sensitivity of the index change to the exact shape of the exciton peak this result is in good agreement with our calculations.

Figure 1(b) shows the absorption spectra and  $\Delta n$  for  $\hat{e} \perp$  plane.  $\Delta n$  is qualitatively similar to that for  $\hat{e} \parallel$  plane except in this polarization there is only one negative peak because only the lh exciton is present. For photon energies below the exciton peak(s)  $\Delta n$  is therefore greater for  $\hat{e} \parallel$  plane than for  $\hat{e} \perp$  plane. This is seen in our calculation and has also been observed experimentally.<sup>4</sup> At 1.42 eV, 50 meV below the zero field hh exciton energy, and at a field of  $6.6 \times 10^4$  V/cm we predict an induced birefringence of  $9.4 \times 10^{-4}$ . This agrees quite well with the value of  $5.2 \times 10^{-4}$  of Ref. 4 and provides strong support for our model. This birefringence may be useful for polarization sensitive devices.

Reasonable requirements to make a useful device are that  $\lambda/2$  change in the optical path length  $nL$  must be obtained, and  $L$  must be less than the absorption length  $1/\alpha$ .  $\Delta n$  reaches its largest value of 0.04 near the exciton peak, corresponding to  $\lambda/2$  change in a path length of  $\sim 11 \mu\text{m}$ ; however, at this wavelength the absorption length is  $\sim 1 \mu\text{m}$ . To get around this problem, we can take advantage of the energy dependence of  $\Delta n$  and  $\alpha$ .  $\Delta n \propto 1/(\Delta E)^2$ , whereas from Urbach's rule  $\alpha = \alpha_0 \exp(-\Delta E/E_0)$  decreases more rapidly with decreasing energy. For energies sufficiently far

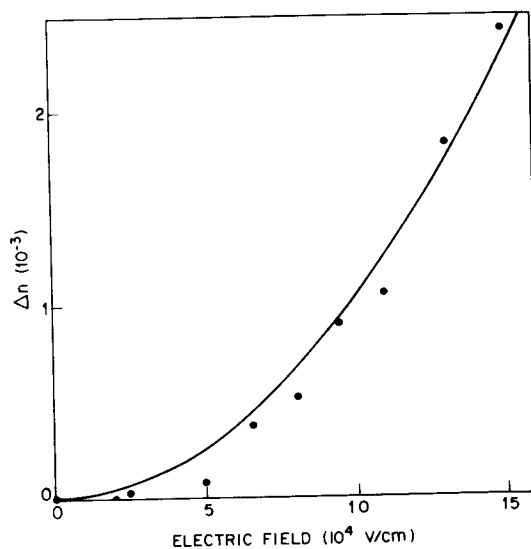


FIG. 2. Refractive index change as a function of applied field at an incident photon energy of 1.4 eV. The incident polarization was perpendicular to the plane of the layers. The solid curve represents the functional form  $\Delta n = (-1/2)n^3sE^2$  with  $-(1/2)n^3s = 10^{-13} \text{ cm}^2/\text{V}^2$ .

below the exciton energy the residual absorption will be small enough to construct a useful device. We take  $\alpha_0$  to be the band-to-band absorption coefficient of  $5000 \text{ cm}^{-1}$ .  $E_0$  has been measured to be 6–8 meV in MQW's<sup>11</sup> and bulk GaAs<sup>12,13</sup> at zero field. From photoconductivity measurements, however, we have measured  $E_0$  to be 11 meV at low field. We will use the latter as a worst case estimate. For  $\hat{e}_\perp$  plane at 1.4 eV and taking  $\Delta E = 60 \text{ meV}$ ,  $\alpha \approx 21 \text{ cm}^{-1}$ , corresponding to an absorption length of  $470 \mu\text{m}$ . Figure 2 shows  $\Delta n$  versus electric field at incident photon energy of 1.4 eV with  $\hat{e}_\perp$  plane. Sufficient change in optical path length occurs at a field of  $\sim 10^5 \text{ V/cm}$ . This length compares quite favorably with the typical LiNbO<sub>3</sub> device length of  $750 \mu\text{m}$ .<sup>14</sup> The solid curve in Fig. 2 is a fit of the form  $\Delta n = -(1/2)n^3sE^2$  with  $(-1/2)n^3s = 10^{-13} \text{ cm}^2/\text{V}^2$ . This dependence has been observed by Glick *et al.*<sup>4</sup> and agrees with the underlying quadratic dependence of the QCSE. Since the index change in LiNbO<sub>3</sub> is due to the linear electro-optic effect, at higher fields the QW device will have even more favorable performance. The required length may be even shorter, depending on the actual value of  $\alpha$  in the band tail. The residual absorption does make it difficult to chain multiple devices together however, so it is important that this be measured.

An important aspect of the use of the QCSE as an absorption modulator is whether it introduces significant chirp. Chirp in a laser device is often parametrized by  $\alpha_c \equiv \Delta n'/\Delta n''$  where the refractive index  $n \equiv n' + in''$  and the changes  $\Delta n'$  and  $\Delta n''$  result from changes in carrier density. An analogous parameter in our case is  $\alpha_{\text{ceff}} \equiv \Delta n'/\Delta n''$  where the changes  $\Delta n'$  and  $\Delta n''$  are induced by field rather than population. For the data in Fig. 1(a), if we operate at  $\sim 1.45 \text{ eV}$  where absorption change with field is largest, we estimate  $\alpha_{\text{ceff}} \sim 0.25$ . For photon energy  $\sim 1.445 \text{ eV}$  where  $\Delta\alpha \sim 1000 \text{ cm}^{-1}$ , we estimate  $\alpha_{\text{ceff}} \sim 1.5$ . These numbers compare well with typical  $\alpha_c$  values ( $\lesssim 5$ ) for GaAs lasers,<sup>15</sup> showing that QCSE absorption modulators offer relatively very low chirp. It should be noted, however, that in addition to the effect of chirp, the bandwidth will also be increased by the nonlinear response of the device.

Our calculation differs from the previous purely theoretical calculations<sup>5,6</sup> in several ways. Our calculation is based on experimental spectra and does not rely on simplified treatments of excitons.<sup>5</sup> More important, ours includes excitonic effects (including those *above* the band gap) and both lh and hh hole contributions, and is calculated for both TE and TM polarizations. The previous calculations do not include conservation of area and hence will tend to overestimate the refractive index changes. The previous calculation was also performed for relatively wide wells at high fields. It has been shown theoretically that, for a given field, the tran-

sition from QCSE-like to bulk-like Franz-Keldysh electroabsorption was very rapid with increasing well width.<sup>16</sup> For the field and width previously considered,<sup>6</sup> the resulting electroabsorption should be almost indistinguishable from the Franz-Keldysh effect. Consequently, except for small peaks near the original subband absorption edges, the changes in refractive index should be little different from those in bulk material showing the usual Franz-Keldysh effect. In our case, however, we are still dominated by quantum-confined behavior because we consider narrower wells, and hence our calculation should better exemplify the special features of refractive index changes associated with quantum-confined behavior.

In conclusion, we have calculated the refractive index changes associated with the QCSE, using experimental electroabsorption data, together with optical sum rules. We find good agreement with published experimental results. The calculations show that electro-optic devices could be made using this effect, and that the electroabsorptive modulators are capable of very low chirp modulation.

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