Performance Analysis of Linear Binary Block Coded Optical PPM CDMA Systems With Soft-Decision Decoding

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Recently, optical code-division multiple-access (CDMA) schemes have attracted much attention particularly in the field of fiber-optic networks, because they allow multiple users to access the network asynchronously and simultaneously. In optical CDMA systems, multi-user interference is one of the most serious problems. To improve performance of optical CDMA systems, some researchers have applied error-correction codes, such as convolutional codes, Reed Solomon codes, and BCH codes [1][2]. It has been well-known that soft-decision decoding of these codes can achieve better performance than hard-decision decoding [3]. However, performance of linear binary block coded optical pulse position modulation (PPM) CDMA systems with soft-decision decoding has not been clarified.

In this paper, we obtain upper bounds on the bit-error rate (BER) for performance of linear binary block coded optical PPM CDMA systems with soft-decision decoding. We use a union bounding technique to obtain these bounds, so our results correspond to the upper bounds. We consider systems using binary-PPM (BPPM) and an avalanche photodiode (APD), and treat APD noise, thermal noise, and multi-user interference using a Gaussian approximation. We show that linear block-coded optical PPM CDMA systems using soft-decision decoding can achieve low BER performance and exhibit no error floor in the range of the practical BER.

Direct-detection optical CDMA systems require $\{0,1\}$-valued code sequences with good correlation properties: small cross-correlations between the sequences are essential for low multi-user interference. As our signature code sequences, we employ low-weight optical orthogonal codes (OOC's) with out-of-phase-auto- and cross-correlations bounded by one. We assume each user to

be assigned an optical code sequence of OOC's with length F and weight K. For OOC with length F, weight K, and the maximum values of out-of-phase-auto- and cross-correlations equal to one, the number of code sequences is upper bounded by $N \leq \left\lfloor \frac{F-1}{K(K-1)} \right\rfloor$.

We consider an (n,k) code where a block of k information bits is mapped into a code word of length n selected from the set of $M = 2^k$ code words. We also consider binary PPM (BPPM). A laser pulse is emitted on the 0th or first slot according to a coded bit. The output laser pulse is converted into the assigned optical code sequence, that is, the signature code sequence by the sequence encoder comprising a set of optical fiber delay lines. Then, the pulses spread over the slot are transmitted over the fiber to the desired destination.

For optical BPPM CDMA systems, the optimum receiver can be realized by means of two matched filters, one matched to the code sequence corresponding to a transmission of a 0, and the other to the code sequence corresponding to a transmission of 1, followed by a decoder that forms the M correlation metrics corresponding to the M possible code words. The matched filter can be realized by an optical correlator that is a set of optical delay lines inversely matched to the pulse spacings. The optical intensity received at the positions of '1' of the code sequence for the desired channel is summed up at the last chip. The output of the optical correlator is converted into the electrical signal by the APD. The accumulated output during each chip interval T_c is assumed to be a Gaussian random variable. The performance is analyzed in the chip-synchronous case and thus the performance results in the upper bounds on that of the system. Let Y_{0j} and Y_{1j} denote the input samples to the combiner. The correlation metrics formed by the decoder may be expressed as [3]

$$CM_i = \sum_{j=1}^{n} [b_{ij} Y_{1j} + (1 - b_{ij}) Y_{0j}], \qquad i = 1, 2, \cdots, M \tag{1}$$

where b_{ij} represents the jth bit in the ith code word. The receiver selects the code word corresponding to the largest of the $\{CM_i\}$ as the transmitted code word. Suppose the all-zero code word is transmitted. Using a union bound and the minimum distance, the conditional probability that $\Pr\{CM_m > CM_1 | 0, \boldsymbol{\kappa}_0 = \mathbf{o}, \boldsymbol{\kappa}_1 = \boldsymbol{l}\}$ is derived as

$$\Pr\{CM_m > CM_1 | 0, \boldsymbol{\kappa}_0 = \mathbf{o}, \boldsymbol{\kappa}_1 = \boldsymbol{l}\} = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma_m^2}} e^{-(x-\mu_m)^2/2\sigma_m^2} \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi\sigma_0^2}} e^{-(y-\mu_0)^2/2\sigma_0^2} dy dx. \tag{2}$$

Here, $\mathbf{o} = (0, 0, \cdots, 0)$, $\boldsymbol{l} = (l_0, l_1, \cdots, l_n)$, and $g(\boldsymbol{l}) = \sum_{i=1}^{n} l_i$. The means, μ_1 and μ_m, and the variances, σ_1^2 and σ_m^2 of CN_m and CM_1, are derived, respectively as follows:

$$\mu_1 = n \left\{ G T_c [K\lambda_s + (KN - K)\lambda_s / M_e + K\lambda_b + I_b/e] + T_c I_s / e \right\}, \tag{3}$$

$$\sigma_1^2 = n\left\{G^2F_eT_c[K\lambda_s + (KN-K)\lambda_s/M_e + K\lambda_b + I_b/e] + T_cI_s/e + \sigma_{th}^2\right\}, \quad (4)$$

$$\mu_m = GT_c[g(\boldsymbol{l})\lambda_s + (KN - g(\boldsymbol{l}))\lambda_s/M_e + nK\lambda_b + nI_b/e] + nT_cI_s/e$$
$$+(n-w_m)\left\{GT_c[K\lambda_s + (KN-K)\lambda_s/M_e + K\lambda_b + I_b/e] + T_cI_s/e\right\}, \quad (5)$$

$$\sigma_m^2 = G^2F_eT_c[g(\boldsymbol{l})\lambda_s + (KN - g(\boldsymbol{l}))\lambda_s/M_e + nK\lambda_b + nI_b/e] + nT_cI_s/e + n\sigma_{th}^2$$
$$+(n-w_m)\left\{G^2F_eT_c[K\lambda_s + (KN-K)\lambda_s/M_e + K\lambda_b + I_b/e] + T_cI_s/e + \sigma_{th}^2\right\}. \quad (6)$$

Here, w_m is the weight of the code word having the correlation metric CM_m, G is the average APD gain, I_b is the bulk leakage current, I_s is the APD surface leakage current, F_e is the excess noise factor given by

$$F_e = k_{eff}G + (2 - 1/G)(1 - k_{eff}) \quad (7)$$

where k_{eff} is the APD effective ionization ratio, and σ_{th}^2 is the variance of thermal noise written as

$$\sigma_{th}^2 = 2k_BT_rT_c/(e^2R_L) \quad (8)$$

where k_B is Boltzmann's constant, T_r is the receiver noise temperature, and R_L is the receiver load resistor. Thus, the BER can be bounded by

$$P_b \le \sum_{\boldsymbol{l}} \binom{N-1}{l_1}\binom{N-1}{l_2}\cdots\binom{N-1}{l_d}\left(\frac{K^2}{2F}\right)^{g(\boldsymbol{l})}\left(1-\frac{K^2}{2F}\right)^{d(N-1)-g(\boldsymbol{l})}$$
$$\cdot \Pr\{CM_m > CM_1 | 0, \boldsymbol{\kappa}_0 = \mathbf{0}, \boldsymbol{\kappa}_1 = \boldsymbol{l}\}. \quad (9)$$

We use typical APD parameters in Table 1 in the numerical calculation to be presented in this section. Note that P_W denotes the unit received laser power in a delay-line of optical correlator.

Fig. 1 shows the BER versus the received laser power P_W where the number of users is N=10, OOC with $F = 61$ and K=3, and bit rate is R_b=10 Mbps. Note that when the number of users is larger than the code weight, i.e., $N > K$, an error floor exists in the performance of uncoded PPM CDMA systems. We can see that BCH-coded optical PPM CDMA systems using soft-decision decoding have low BER performance and exhibit no error floor in the range of the practical BER. We can also see that as n increases with keeping the code rate constant, the BER performance can be improved.

Fig. 2 shows the BER versus the number of users N where the received laser power $P_W = -80$ dBW, OOC with $F = 61$ and K=3, and bit rate is R_b=10 Mbps. Note that the uncoded optical PPM CDMA systems cannot accommodate more users than the value of weight of the code sequence used in the system, even with large received laser power. We can see that BCH-coded optical PPM

Table 1: Link parameters

Name	Symbol	Value
Laser wavelength		1.55 μm
APD quantum efficiency	η	0.6
APD gain	G	100
APD effective ionization ratio	k_{eff}	0.02
APD bulk leakage current	I_b	0.1 nA
APD surface leakage current	I_s	10 nA
Modulation extinction ratio	M_e	100

CDMA systems using soft-decision decoding have low BER performance and can accommodate users more than the value of the code weight even when the received laser power is small.

References

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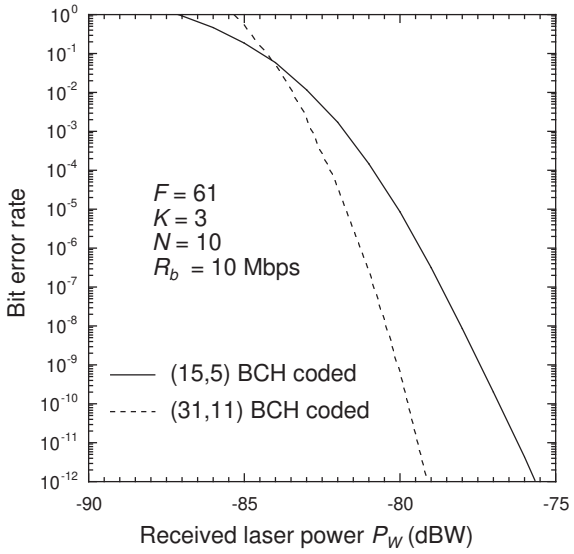

Figure 1: Bit error rate versus received laser power P_W where the bit rate R_b=10 Mbps, the number of users N=10, and OOC with $F = 61$ and K=3.

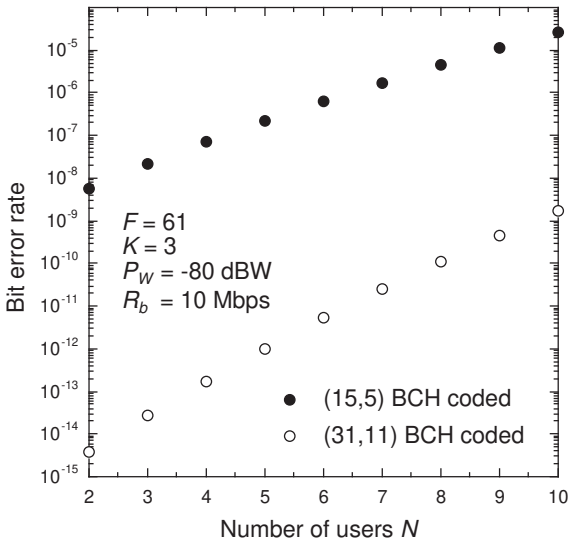

Figure 2: Bit error rate versus the number of users N where the received laser power P_W=-80 dBW, the bit rate R_b=10 Mbps, and OOC with $F = 61$ and K=3.