Joint Design of a Channel-Optimized Quantizer and Multicarrier Modulation

Keang-Po Ho and Joseph M. Kahn

***Abstract*— We present an algorithm for design of a joint source-channel coder using a channel-optimized quantizer and multicarrier modulation. By changing the power of each subchannel in the multicarrier modulation system, different degrees of error protection can be provided for different bits according to their importance. The algorithm converges to a locally optimum system design. Compared to a Lloyd–Max scalar quantizer or a LBG vector quantizer using single-channel transmission, our optimized code can yield substantial performance improvements. The performance improvements are most pronounced at low channel signal-to-noise ratios.**

***Index Terms*— Combined source-channel coding, multicarrier modulation, vector quantization.**

I. Introduction

TRADITIONALLY, source and channel codes are designed separately and then cascaded together, provided an infinite degree of complexity, source, and channel coding can be performed separately without sacrificing fidelity [1]. However, when many source-coding techniques are employed, some coded bits (e.g., the most significant bit of a scalar-quantized signal) are far more important than others. Therefore, the source and channel codes can be combined into a single code in which the objective is to minimize the overall distortion between the original source at the transmitter and its reconstruction at the receiver.

Most studies of combined source-channel coding for continuous-amplitude sources lead to the study of channel-optimized quantizers (COQ's) for noisy channels [2]–[6]. The design algorithms of COQ optimize the quantizer under the assumption of a noisy channel, usually a binary symmetric channel that has a fixed BER. Although joint optimization of the modulation and quantizer has been proposed previously [7]–[10], the previous schemes are subject to some potential drawbacks. The scheme of [8] employed a suboptimal linear receiver instead of the optimal maximum-likelihood receiver. While the scheme of [7] simply employed identical trellises for both a trellis-coded quantizer (TCQ) and trellis-coded modulation (TCM), the TCQ/TCM joint optimization relied upon complicated techniques, i.e., a quasi-Newton optimization routine [9] and a simulated annealing-based algorithm [10]. In addition, all of the modulation schemes proposed in [7]–[10] may be difficult to implement. Our algorithm here is similar in concept to those in [7]–[10].

This letter extends our previous work [11] on a combined source-channel coding scheme using multicarrier modulation (MCM). While previous work used source-optimized MCM, we iteratively optimize a COQ and MCM here. MCM [12] provides the flexibility to change the power, modulation, and channel encoding of each individual subchannel, so that different degrees of error protection may be provided for different bits according to their importance. The overall distortion can be thereby minimized. MCM has been employed in asymmetrical digital subscriber lines (ADSL) [13], and in digital audio and television broadcasting [14], [15]. While the power-allocation algorithm considered here has been proposed for weighted PCM systems [16] and the COQ has been studied extensively in [2]–[6], we believe that the present work is the first to apply a power-allocation algorithm to MCM for a COQ.

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II. Overall Distortion of the System

Fig. 1 represents the schematic diagram of a communication system. The first stage is a vector quantizer. The vector quantizer q with Q levels is defined by a finite set of *quantization levels* $\mathbf{y}_i$, together with the partition of the input space into sets S_i such that $q(\mathbf{x}) = \mathbf{y}_i$, if $\mathbf{x} \in S_i$, $i = 1, 2, \cdots, Q$, where $\mathbf{x}$ represents the k-dimensional source vector. A scalar quantizer is a special case of a vector quantizer having unit dimension ($k = 1$).

After quantization, a binary mapping maps the quantization levels into fixed-length binary codewords, $b_i = b(\mathbf{y}_i)$, where $b_i = 0, 1, \cdots, M-1$ are the n-bit integer values of the binary codewords, and $M = 2^n$. Below, we represent the cascade of the quantizer and encoder as $\gamma(\cdot) = b(q(\cdot))$. The binary codewords will be conveyed by a suitable channel modulation scheme. Because of channel-induced noise, there is a nonzero transition probability $p_{j|b_i}$ that the demodulated output j does not equal the modulated input b_i. The receiver decodes channel output j into $g(j) = \mathbf{r}_j$, which can take one of the M possible values $\mathbf{r}_0, \mathbf{r}_1, \cdots, \mathbf{r}_{M-1}$, the decoder *reconstruction levels.* Note that, in general, M need not be equal to Q. Even if $M = Q$, the reconstruction levels $\mathbf{r}_j$ are not necessarily equal to the quantization levels $\mathbf{y}_i$.

The overall mean squared-error (MSE) incurred in the system is

$$D(q, b, p_{j|b_i}, g) = \sum_{j=0}^{M-1} \sum_{i=1}^{Q} p_{j|b_i} \int_{S_i} ||\mathbf{x} - \mathbf{r}_j||^2 p(\mathbf{x})\, d\mathbf{x}. \tag{1}$$

We would like to minimize the overall MSE (1) by appropriate choices of q, b, g, and $p_{j|b_i}$.

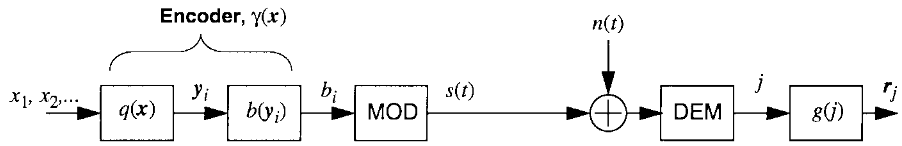

Fig. 1. Schematic diagram of a data-transmission system for an analog discrete-time source. At the transmitter, the source is quantized by either a scalar or vector quantizer to codewords, which are mapped to bits that drive a modulator. The receiver demodulates the signal to bits and returns the codewords. (MOD: modulator; DEM: demodulator.)

MCM will be used in the channel coding [11]. Each subchannel of MCM can utilize different power, channel encoding, and carrier $\varphi_m(t)$, so that different error protection can be provided for different bits. The most popular MCM system uses tones at different frequencies as carriers [12]–[15]. Alternatively, a time-division multiplexed system using different constellations in each time slot can also be considered as "generalized" MCM [11].

We will use $N = n$ subchannels to transmit the n-bit signal, assigning different bits to different subchannels in consecutive order. The BER p_{b_m} is a function of E_m, the power in the mth subchannel, i.e., $p_{b_m} = P_m(E_m)$. The sum of these powers is equal to the total power E_T. Usually the BER's are small, so that the probability of multiple bit errors within the same binary codeword is very small, which simplifies the transition probabilities to [11]

$$p_{j|i} \cong \begin{cases} 1 - \sum_{m=1}^{n} p_{b_m}, & i = j \\ p_{b_m}, & l_m(i, j) = 1,\, d_H(i, j) = 1 \\ 0, & d_H(i, j) > 1 \end{cases} \tag{2}$$

where $l_m(i, j) = 1$ if the binary codewords represented by i and j differ in the mth position, and otherwise $l_m(i, j) = 0$. The Hamming distance $d_H(i, j)$ between codewords i and j is the sum of $l_m(i, j)$. We show below that the approximation in (2) is valid for the combined source-channel coding scheme considered in this letter.

The MSE (1) does not depend on the values of quantization levels $\mathbf{y}_i$. If we define $\mathbf{y}_i$ as the centroid of the respective partition, after some algebra the overall MSE can be decomposed into the sum of the quantization distortion (the first term) and the channel distortion (the second term):

$$D(q, b, p_{j|b_i}, g) = \sum_{i=1}^{Q} \int_{S_i} \|\mathbf{x} - \mathbf{y}_i\|^2 \, p(\mathbf{x})\, d\mathbf{x} + \sum_{j=0}^{M-1} \sum_{i=1}^{Q} P_{j|b_i} (\mathbf{y}_i - \mathbf{r}_j)^2 p_{y_i} \tag{3}$$

where p_{y_i} is the *a priori* probability of quantization level $\mathbf{y}_i$. The quantization distortion is the smallest possible squared-error distortion attainable by the partition S_i.

With the approximation (2), the channel distortion in (3) can be rewritten as

$$D_C(q, b, p_{j|b_i}, g) = \sum_{i=1}^{Q} p_{y_i} \|\mathbf{r}_{b_i} - \mathbf{y}_i\|^2 + \sum_{m=1}^{n} W_m P_m(E_m) \tag{4}$$

where W_m is the weighting factor of the mth bit

$$W_m = \sum_{i=1}^{Q} p_{y_i} [\|\mathbf{r}_{i_{e,m}} - \mathbf{y}_i\|^2 - \|\mathbf{r}_{b_i} - \mathbf{y}_i\|^2] \tag{5}$$

and $i_{e,m} = b_i + (1 - 2i_m) 2^{n-m}$, where i_m is the mth bit of b_i, i.e., $b_i = \sum_{m=1}^{n} i_m 2^{n-m}$.

To minimize the overall distortion, the power can be allocated by numerically solving [11]

$$W_m \frac{dP_m(E_m)}{dE_m} = \lambda, \qquad m = 1, 2, \cdots n$$
$$\sum_{m=1}^{n} E_m = E_T. \tag{6}$$

III. Optimal System Design and Algorithm

The COQ/MCM is jointly optimized in three stages. First, for a fixed power allocation and a fixed decoder, we optimize the encoder γ. Second, for a fixed encoder γ and fixed power allocation, we optimize the decoder g. Third, for a fixed encoder γ and decoder g, we allocate the power of the MCM system. The final system will fulfill all optimality conditions for the COQ-MCM. This approach has one step (for power allocation) more than those for COQ [2]–[5]. The similar three-step approach was used in [8], [9] for joint COQ and modulation optimization. The following iterative algorithm determines the optimal combined source-channel code for a fixed overall power of E_T and number of bits n.

1) Set $k = 0$ (k is the iteration index). Choose initial reconstruction levels $\mathbf{r}_i^{(0)}$ and $b_i^{(0)}$. If initial weight factors $W_m^{(0)}$ are available, use them to find the corresponding initial $p_{j|i}^{(0)}$; otherwise, simply choose $p_{j|i}^{(0)} = \delta_{ij}$. Set $D^{(0)} = \infty$. Also choose ϵ, a small positive number, for the stopping condition.
2) For fixed $\mathbf{r}_j^{(k)}$ and $p_{j|i}^{(k)}$, obtain the new encoder $\gamma^{(k)}$ by choosing the partition [3]–[6]

$$S_i = \Bigg\{ \mathbf{x} : \sum_{j=0}^{M-1} p_{j|b_i} \|\mathbf{x} - \mathbf{r}_j\|^2 \le \cdot \sum_{j=0}^{M-1} p_{j|b_k} \|\mathbf{x} - \mathbf{r}_j\|^2,\, k = 1, \cdots, Q \Bigg\}. \tag{7}$$

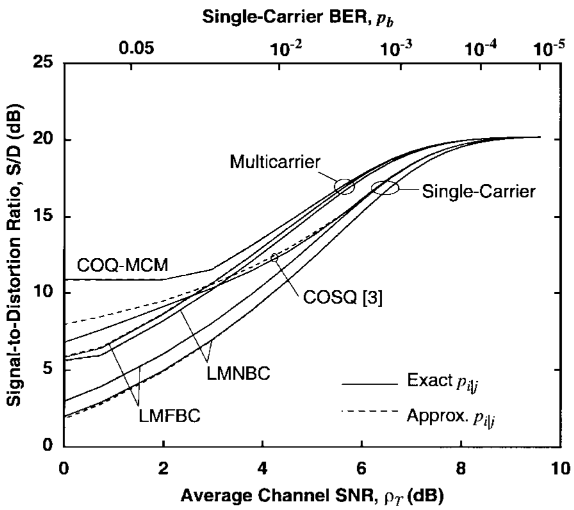

Fig. 2. Performance comparison of a four-bit scalar quantizer for a Gaussian source, where the quantized output is transmitted via single-carrier or multicarrier channels. The encoder, MCM, and decoder are jointly optimized using the algorithm in Section III. For comparison, Lloyd–Max-quantized data are transmitted over single-carrier or multicarrier systems, and the COSQ [3] is also utilized.

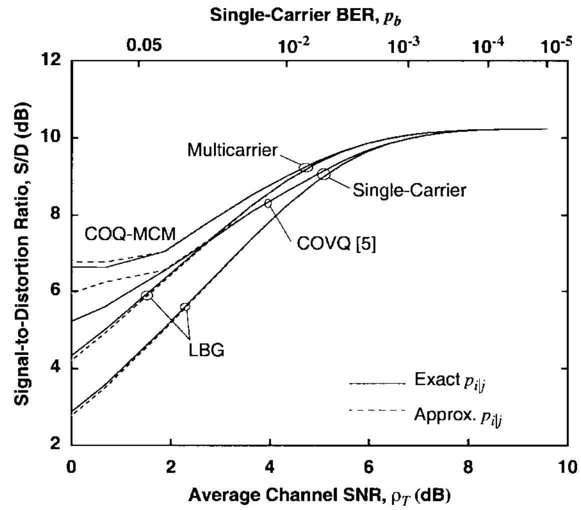

Fig. 3. Performance comparison of four-dimensional vector quantizers for a Markov Gaussian source. The quantized output is transmitted via single-carrier or multicarrier channels. The encoder, MCM, and decoder are jointly optimized using the algorithm in Section III. For comparison, LBG quantized data are transmitted over single-carrier or multicarrier systems, and the COVQ [5] is also utilized.

3) Set $k = k + 1$. For fixed $S_i^{(k-1)}$ and $p_{j|i}^{(k-1)}$, find the decoder $g^{(k)}$ by evaluating $\mathbf{r}_j^{(k)}$ [3]–[5]

$$\mathbf{r}_j = \frac{\sum_{i=1}^{Q} p_{j|b_i} \int_{S_i} \mathbf{x} p(\mathbf{x})\, d\mathbf{x}}{\sum_{i=1}^{Q} p_{j|b_i} \int_{S_i} p(\mathbf{x})\, d\mathbf{x}}, \qquad j = 1, 2, \cdots, M. \quad (8)$$

4) Evaluate the weighting factors $W_m^{(k)}$ using (5). Find the optimal power allocation $E_m^{(k)}$ and the BER's of the subchannels $p_{b_m}^{(k)}$ by solving (6). Obtain the transition probability matrix $p_{j|i}^{(k)}$ for fixed $\mathbf{r}_j^{(k)}$ and $S_i^{(k-1)}$.
5) Compute the MSE $D^{(k)}$ associated with $\gamma^{(k-1)}$, $p_{j|i}^{(k)}$, and $g^{(k)}$ using (3).
6) If $(D^{(k-1)} - D^{(k)})/D^{(k)} < \epsilon$, the iteration is finished; otherwise, go to Step 2.

Because the overall distortion decreases at each step, the algorithm will converge, but usually converges to a *locally* optimized solution.

IV. Numerical Examples

We assume that the source is Gaussian-distributed and that the channel is an additive white Gaussian noise (AWGN) channel. Without loss of generality, all subchannels use binary phase shift keying (BPSK), which has a BER given by $p_b = \frac{1}{2}\text{erfc}(\sqrt{\rho})$, where erfc$(\cdot)$ is the complementary error function and ρ is the signal-to-noise ratio (SNR) of the channel.

Figs. 2 and 3 show the signal-to-distortion ratio (S/D) of a source-channel coder iteratively optimized using the algorithm in Section III for four-bit scalar and vector quantizers, respectively. These figures also show the channel-optimized scalar quantizer (COSQ) [3] and the channel-optimized vector quantizer (COVQ) [5] for comparison, respectively. Because the algorithm in Section III is based on the simplified transition probabilities (2), after the locally optimized result is obtained, we have further optimized the performance by using the exact transition probabilities [11], omitting Step 4. The further optimized curves are plotted in Figs. 2 and 3 as the solid curves. However, numerical results show no observable difference between the results obtained by the simplified and the exact transition probabilities.

In Fig. 2, the LMFBC and LMNBC results have used the Lloyd–Max quantizer [17], [18] with natural binary coding (NBC) or folded binary coding (FBC) [3]. For a very noisy channel, the improvement using our algorithm is substantial. For example, for a single-carrier BER of $p_b = 10^{-2}$ (corresponding to average channel SNR ρ_T = 4.32 dB), the improvement in S/D by using our algorithm compared with the single-carrier modulation with LMFBC is 2.93 dB.

Fig. 3 shows the S/D obtained using the algorithm of Section III for a four-dimensional vector quantizer for a first-order Markov-Gaussian source having a correlation coefficient of 0.9. The quantization rate is 1 bit/sample. Fig. 3 also shows the performance of a LBG quantizer [19] designed by the "splitting" algorithm [20] using a training sequence. The same LBG quantizer is also used as the initial codebook for COVQ and our COQ-MCM code. For very noisy channels, the improvement using our algorithm is significant. For example, at a single-carrier BER of $p_b = 10^{-2}$, the improvement in S/D obtained using our algorithm compared with the single-carrier scheme is 0.77 dB.

In both Figs. 2 and 3, the improvement of COQ-MCM increases for noisier channels. The results of [11] are indicated by multicarrier-LMNBC, multicarrier-LMFBC, and multicarrier-LBG for comparison.

V. Conclusion

We have studied the problem of combined quantizer and modulation design when the quantizer outputs are to be transmitted via noisy channels. The combined source-channel coding scheme uses MCM to provide unequal error protection to different bits according to their importance. An iterative algorithm has been developed for obtaining a locally optimal encoder/modulation/decoder combination. Numerical results shows the this design technique offers improvements over the Lloyd–Max scalar quantizer, LBG vector quantizer, and the channel-optimized quantizer. The improvement is more pronounced for noisier channels.

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