Free-Space Optical Communication Through Atmospheric Turbulence Channels

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***Abstract*—In free-space optical communication links, atmospheric turbulence causes fluctuations in both the intensity and the phase of the received light signal, impairing link performance. In this paper, we describe several communication techniques to mitigate turbulence-induced intensity fluctuations, i.e., signal fading. These techniques are applicable in the regime in which the receiver aperture is smaller than the correlation length of fading and the observation interval is shorter than the correlation time of fading. We assume that the receiver has no knowledge of the instantaneous fading state. When the receiver knows only the marginal statistics of the fading, a symbol-by-symbol ML detector can be used to improve detection performance. If the receiver has knowledge of the joint temporal statistics of the fading, maximum-likelihood sequence detection (MLSD) can be employed, yielding a further performance improvement, but at the cost of very high complexity. Spatial diversity reception with multiple receivers can also be used to overcome turbulence-induced fading. We describe the use of ML detection in spatial diversity reception to reduce the diversity gain penalty caused by correlation between the fading at different receivers. In a companion paper, we describe two reduced-complexity implementations of the MLSD, which make use of a single-step Markov chain model for the fading correlation in conjunction with per-survivor processing.**

***Index Terms*—Atmospheric turbulence, free-space optical communication, MLSD, spatial diversity reception.**

I. Introduction

FREE-SPACE optical communication has attracted considerable attention recently for a variety of applications [1]–[4]. Because of the complexity associated with phase or frequency modulation, current free-space optical communication systems typically use intensity modulation with direct detection (IM/DD). Atmospheric turbulence can degrade the performance of free-space optical links, particularly over ranges of the order of 1 km or longer. Inhomogeneities in the temperature and pressure of the atmosphere lead to variations of the refractive index along the transmission path. These index inhomogeneities can deteriorate the quality of the received image and can cause fluctuations in both the intensity and the phase of the received signal. These fluctuations can lead to an increase in the link error probability, limiting the performance of communication systems. Aerosol scattering effects caused by rain, snow and fog can also degrade the performance of free-space optical communication systems [5], [6], but are not treated in this paper.

Atmospheric turbulence has been studied extensively and various theoretical models have been proposed to describe turbulence-induced image degradation and intensity fluctuations (i.e., signal fading) [7]–[13]. Two useful parameters describing turbulence-induced fading are d_0, the correlation length of intensity fluctuations and τ_0, the correlation time of intensity fluctuations. When the receiver aperture D_0 can be made larger than the correlation length d_0, then turbulence-induced fading can be reduced substantially by aperture averaging [13].

Because it is not always possible to satisfy the condition $D_0 > d_0$, in this paper, we propose alternative techniques for mitigating fading in the regime where $D_0 < d_0$. At the bit rates of interest in most free-space optical systems, the receiver observation interval T_0 during each bit interval is smaller than the turbulence correlation time τ_0. Throughout this paper, we will assume that $D_0 < d_0$ and $T_0 < \tau_0$. The techniques we consider are based on the statistical properties of turbulence-induced signal intensity fading, as functions of both temporal and spatial coordinates. Our approaches can be divided into two categories: temporal-domain techniques and spatial-domain techniques.

In the temporal-domain techniques, one employs a single receiver. If the receiver has knowledge of the marginal fading distribution, but knows neither the temporal fading correlation nor the instantaneous fading state, a maximum-likelihood (ML) symbol-by-symbol detection technique can be used. If the receiver further knows the joint temporal fading distribution, but not the instantaneous fading state, the receiver can use a ML sequence detection (MLSD) technique to mitigate turbulence-induced fading.

In the spatial-domain techniques, one must employ at least two receivers to collect the signal light at different positions or from different spatial angles. To maximize the diversity reception gain, the multiple receivers should be placed as far apart as possible, so that the turbulence-induced fading is uncorrelated at the various receivers. In practice, however, it may not always be possible to place the receivers sufficiently far apart. Hence, in this paper, making use of the spatial correlation of turbulence-induced fading, we derive the optimal ML detection scheme for correlated spatial diversity reception.

The remainder of this paper is organized as follows. In Section II, we review the theories used to model atmospheric turbulence and use them to derive the spatial and temporal coherence properties of the optical field in weak turbulence chan-

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nels. We then use the coherence properties introduced to derive the joint probability distribution of the light intensity received on turbulence channels, characterizing the correlation of signal fading over both time and space. In Section III we describe the temporal domain techniques. We first use the marginal distribution of fading to derive a ML symbol-by-symbol detector for systems using on-off keying (OOK). We then use the joint temporal distribution of fading to derive a MLSD for OOK. In Section IV, we introduce the spatial-diversity reception scheme, where the instantaneous fading state at each receiver is unknown. With knowledge of the joint spatial distribution of fading, ML detection can help to mitigate the diversity reception gain penalty caused by correlation among multiple receivers. We also numerically demonstrate that in the dual-receiver case, this ML detection scheme has better performance than conventional equal-gain combining (EGC). We present our conclusions in Section V.

As we show in Section III, the high computational complexity of the MLSD makes it impractical for most applications. In a companion paper [14], we describe two reduced-complexity implementations of the MLSD, which make use of a single-step Markov chain model for the fading correlation in conjunction with per-survivor processing.

II. Optical Communication Through Atmospheric Turbulence

In this section, we first review the theories used to model atmospheric turbulence. We then use these to derive the spatial and temporal coherence properties of the optical field in weak turbulence channels. Finally, we present the joint spatial and temporal distribution of turbulence-induced fading.

A. Modeling of Atmospheric Turbulence

Atmospheric turbulence can be physically described by Kolmogorov theory [7]–[10]. The energy of large eddies is redistributed without loss to eddies of decreasing size until finally dissipated by viscosity. The size of turbulence eddies normally ranges from a few millimeters to a few meters, denoted as the inner scale l_0 and the outer scale L_0, respectively. We can express the refractive index as $n(\overrightarrow{r},t) = n_0 + n_1(\overrightarrow{r},t)$, where n_0 is the average index and n_1 is the fluctuation component induced by spatial variations of temperature and pressure in the atmosphere. The correlation function of n_1 is defined as

$$\Gamma_{n_1}\left(\overrightarrow{r}_1,t_1;\overrightarrow{r}_2,t_2\right) = E\left[n_1\left(\overrightarrow{r}_1,t_1\right)\cdot n_1\left(\overrightarrow{r}_2,t_2\right)\right]. \quad (1)$$

Setting $t_1 = t_2$ in (1), we obtain $\Gamma_{n_1}(\overrightarrow{r}_1,\overrightarrow{r}_2)$. which describes the spatial coherence of the refractive index.

To study the spatial coherence of the refractive index, many models have been proposed, which assume exponential, Gaussian or other solvable function forms for $\Gamma_{n_1}(\overrightarrow{r}_1,\overrightarrow{r}_2)$. We define the wavenumber spectrum $\Phi_n(k)$ to be the spatial Fourier transform of $\Gamma_{n_1}(\overrightarrow{r}_1,\overrightarrow{r}_2)$. A widely used model with good accuracy was proposed by Kolmogorov, which assumes the wavenumber spectrum to be

$$\Phi_n\left(\overrightarrow{k}\right) = 0.033C_n^2 k^{-11/3}. \quad (2)$$

Here, C_n is the wavenumber spectrum structure parameter, which is altitude-dependent. Hufnagel and Stanley gave a simple model for C_n [9]:

$$C_n^2(z) = K_0 z^{-1/3}\exp\left(\frac{-z}{z_0}\right) \quad (3)$$

where K_0 is parameter describing the strength of the turbulence and z_0 is effective height of the turbulent atmosphere. For atmospheric channels near the ground ($z < 18.5$ m), C_n^2 can vary from 10^{-13} m$^{-2/3}$ for strong turbulence 10^{-17} m$^{-2/3}$ to for weak turbulence, with 10^{-15} m$^{-2/3}$ often quoted as a typical "average" value [8]. Other models and recent measurements of the vertical profile of C_n can be found in [11], [12].

B. Spatial and Temporal Coherence of Optical Signals Through Turbulence

To describe spatial coherence of optical waves, the so-called mutual coherence function (MCF) is widely used [8]:

$$\Gamma\left(P_1,t_1;P_2,t_2\right) = E\left[u\left(P_1,t_1\right)\cdot u^*\left(P_2,t_2\right)\right] \quad (4)$$

where $u(P,t)$ is the complex optical field. Setting in $t_1 = t_2$ in (4), we obtain the spatial MCF $\Gamma(\overrightarrow{r}_1,\overrightarrow{r}_2)$. The Rytov method is frequently used to expand the optical field $u(\overrightarrow{r})$:

$$u\left(\overrightarrow{r}\right) = A\left(\overrightarrow{r}\right)\cdot\exp\left[i\phi\left(\overrightarrow{r}\right)\right] = u_0\left(\overrightarrow{r}\right)\cdot\exp\left(\Phi_1\right) \quad (5)$$

where $u_0(\overrightarrow{r})$ is the field amplitude without air turbulence:

$$u_0\left(\overrightarrow{r}\right) = A_0\left(\overrightarrow{r}\right)\cdot\exp\left[i\phi_0\left(\overrightarrow{r}\right)\right]. \quad (6)$$

The exponent of the perturbation factor is:

$$\Phi_1 = \log\left[\frac{A\left(\overrightarrow{r}\right)}{A_0\left(\overrightarrow{r}\right)}\right] + i\left[\phi\left(\overrightarrow{r}\right) - \phi_0\left(\overrightarrow{r}\right)\right] = X + iS \quad (7)$$

where X is the log-amplitude fluctuation and S is the phase fluctuation. We assume X and S be homogeneous, isotropic and independent Gaussian random variables. This assumption is valid for long propagation distances through turbulence.

In order to characterize turbulence-induced fluctuations of the log-amplitude X, we use the log-amplitude covariance function:

$$B_X\left(P_1;P_2\right) = E\left[X\left(P_1\right)X\left(P_2\right)\right] - E\left[X\left(P_1\right)\right]\cdot\left[X\left(P_2\right)\right]. \quad (8)$$

Since the random disturbance is Gaussian-distributed under the assumption of weak turbulence, we can use the Rytov method to derive the normalized log-amplitude covariance function for two positions in a receiving plane perpendicular to the direction of propagation [13]:

$$b_X\left(d_{12}\right) = \frac{B_X\left(P_1,P_2\right)}{B_X\left(P_1,P_1\right)} \quad (9)$$

where d_{12} is the distance between P_1 and P_2. We define the correlation length of intensity fluctuations, d_0, such that $b_X(d_0) = e^{-2}$. When the propagation path length L satisfies the condition $l_0 < \sqrt{\lambda L} < L_0$, where λ is the wavelength and l_0 and L_0 are inner and outer length scales, respectively, d_0 can be approximated by [13]

$$d_0 \approx \sqrt{\lambda L}. \tag{10}$$

In most free-space optical communication systems with visible or infrared lasers and with propagation distance of a few hundred meters to a few kilometers, (10) is valid. We note that aerosol forward scattering can further degrade the coherence of the optical field and thus affect the correlation length. In this paper, however, we focus only on atmospheric turbulence effects.

Atmosphere turbulence also varies with time and leads to intensity fluctuations that are temporally correlated. Modeling the movement of atmospheric eddies is extremely difficult and a simplified "frozen air" model is normally employed, which assumes that a collection of eddies will remain frozen in relation to one another, while the entire collection is translated along some direction by the wind. Taylor's frozen-in hypothesis can be expressed as [10]:

$$n_1\left(\overrightarrow{r},t\right) = n_1\left(\overrightarrow{r} - \overrightarrow{\nu}\,t,0\right) \tag{11}$$

where $\overrightarrow{\nu}$ is the velocity of the wind, which has an average $\overrightarrow{u}$ and a fluctuation $\overrightarrow{\nu}_f$. If $\overrightarrow{\nu}_f$ is negligible and $\overrightarrow{u}$ is transverse to the direction of light propagation, then temporal correlation becomes analogous to spatial correlation; in particular, the correlation time is $\tau_0 = d_0/u$. Assuming a narrow beam propagating over a long distance, the refractive index fluctuations along the direction of propagation will be well-averaged and will be weaker than those along the direction transverse to propagation. Therefore we need only consider the component of the wind velocity vector perpendicular to the propagation direction $u_\perp$. The turbulence correlation time is therefore

$$\tau_0 = \frac{d_0}{u_\perp}. \tag{12}$$

C. Probability Distributions of Turbulence-Induced Intensity Fading

As discussed previously, when the propagation distance is long, log-amplitude fluctuations can become significant. In this section, we will derive the statistical properties of the log-amplitude fluctuations, which we refer to as "intensity fading" or simply "fading." The marginal distribution of fading is derived in Section I, while the joint spatial and temporal distribution of fading are derived in Section II.

1) Marginal Distribution of Fading: In this section, we derive the marginal distribution of fading at a single point in space at a single instant in time. The marginal distribution is used in symbol-by-symbol ML detection, which is discussed in Section III-A.

For propagation distances less than a few kilometers, variations of the log-amplitude are typically much smaller than variations of the phase. Over longer propagation distances, where turbulence becomes more severe, the variation of the log-amplitude can become comparable to that of the phase. Based on the atmosphere turbulence model adopted here and assuming weak turbulence, we can obtain the approximate analytic expression for the covariance of the log-amplitude fluctuation X of plane and spherical waves [10]:

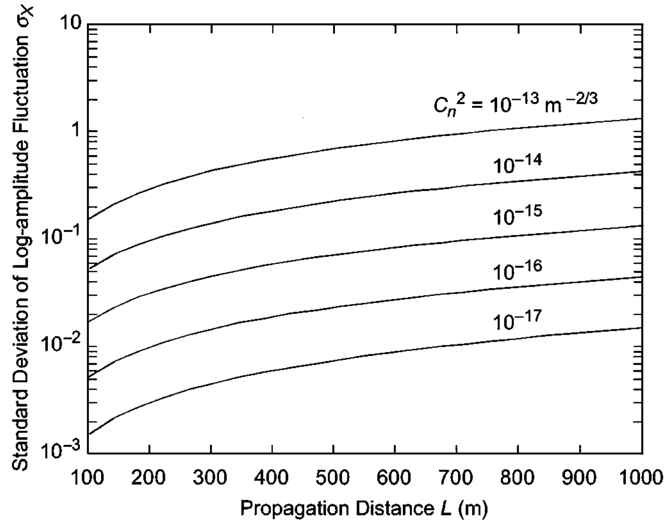


Fig. 1. Standard deviation of the log-amplitude fluctuation versus propagation distance for a plane wave.

$$\sigma_X^2\big|_{\text{plane}} = 0.56\left(\frac{2\pi}{\lambda}\right)^{7/6}\int_0^L C_n^2(x)(L-x)^{5/6}dx \tag{13}$$

$$\sigma_X^2\big|_{\text{spherical}} = 0.56\left(\frac{2\pi}{\lambda}\right)^{7/6}\int_0^L C_n^2(x)\left(\frac{x}{L}\right)^{5/6}(L-x)^{5/6}dx. \tag{14}$$

Fig. 1 shows the standard deviation of the log-amplitude fluctuation σ_X for a plane wave, computed using (13), as a function of the propagation distance L. In Fig. 1, we again assume a wavelength of 529 nm and assume $C_n^2(z)$ to be constant. Fig. 1 shows that for propagation distances of a kilometer, σ_X varies from 10^{-2} to 1 for different values of C_n^2.

Consider the propagation of light through a large number of elements of the atmosphere, each causing an independent, identically distributed phase delay and scattering. By the Central Limit Theorem, the marginal distribution of the log-amplitude is Gaussian:

$$f_X(X) = \frac{1}{(2\pi\sigma_X^2)^{1/2}}\exp\left\{-\frac{(X-E[X])^2}{2\sigma_X^2}\right\}. \tag{15}$$

The light intensity I is related to the log-amplitude X by

$$I = I_0\exp(2X - E[x]). \tag{16}$$

Where $E[X]$ is the ensemble average of log-amplitude X.

From (15) and (16), the average light intensity is:

$$E[I] = E\left[I_0\exp(2X - 2E[X])\right] = I_0\exp\left(2\sigma_X^2\right). \tag{17}$$

Hence, the marginal distribution of light intensity fading induced by turbulence is log-normal:

$$f_I(I) = \frac{1}{2I}\frac{1}{(2\pi\sigma_X^2)^{1/2}}\exp\left\{-\frac{[\ln(I)-\ln(I_0)]^2}{8\sigma_X^2}\right\}. \tag{18}$$

2) Joint Spatial and Temporal Distributions of Fading: In this section, we derive the joint spatial and temporal distributions of fading. The joint spatial distribution describes the fading at multiple points in space at a single instant of time and is used in Section IV in evaluating the performance of spatial diversity reception. The joint temporal distribution describes the fading at a single point in space at multiple instants of time. This distribution is the basis for the MLSD introduced in Section III.

We assume that the log-amplitude at n receivers is described by a joint Gaussian distribution. From (9), the auto-covariance matrix of the log-amplitude at n receivers in a plane transverse to the direction of propagation is given by

$$C_X = \begin{bmatrix} \sigma_X^2 & \sigma_X^2 b_X(d_{12}) & \cdots & \sigma_X^2 b_X(d_{1n}) \\ \sigma_X^2 b_X(d_{21}) & \sigma_X^2 & \cdots & \sigma_X^2 b_X(d_{2n}) \\ \cdots & \cdots & \cdots & \cdots \\ \sigma_X^2 b_X(d_{n1}) & \sigma_X^2 b_X(d_{n2}) & \cdots & \sigma_X^2 \end{bmatrix}_{n\times n} \quad (19)$$

where d_{ij} is the distance between points i and j in the receiver plane. Based on "frozen-in" model and ignoring wind velocity fluctuations, (19) can also be modified to describe temporal fading correlation by making the substitution

$$d_{ij} = |i-j|Tu_\perp \quad (20)$$

where T is the time interval between observations. In all that follows, we assume a communication system using OOK, in which case, T is the bit interval. We denote the covariance matrix of a string of n bits as shown in (21) at the bottom of the page. From the previous discussion about spatial and temporal correlation (see Section II-B), the correlation time τ_0 equals $d_0/u_\perp$.

The joint distribution of intensity for a sequence of transmitted On bits is

$$f_{\overline{I}}(I_1, I_2, \ldots, I_n) = \frac{1}{2^n \prod_{i=1}^n I_i} \frac{1}{(2\pi)^{n/2} |C_X^T|^{1/2}} \cdot \exp\left\{-\frac{1}{8}\left[\ln\left(\frac{I_1}{I_0}\right) \cdots \ln\left(\frac{I_n}{I_0}\right)\right] (C_X^T)^{-1} \begin{bmatrix} \ln\left(\frac{I_1}{I_0}\right) \\ \cdots \\ \ln\left(\frac{I_n}{I_0}\right)\end{bmatrix}\right\}. \quad (22)$$

Similarly, when an On bit is transmitted, the joint distribution of intensity at n receivers is given by

$$f_{\overline{I}}(I_1, I_2, \ldots, I_n) = \frac{1}{2^n \prod_{i=1}^n I_i} \frac{1}{(2\pi)^{n/2} |C_X|^{1/2}} \cdot \exp\left\{-\frac{1}{8}\left[\ln\left(\frac{I_1}{I_0}\right) \cdots \ln\left(\frac{I_n}{I_0}\right)\right] \cdot C_X^{-1} \begin{bmatrix} \ln\left(\frac{I_1}{I_0}\right) \\ \cdots \\ \ln\left(\frac{I_n}{I_0}\right)\end{bmatrix}\right\}. \quad (23)$$

III. ML Detection of On-Off Keying in Turbulence Channels

In this paper, we consider intensity modulation/direct detection (IM/DD) links using on-off keying (OOK). In most practical systems, the receiver signal-to-noise ratio (SNR) is limited by shot noise caused by ambient light much stronger than the desired signal and/or by thermal noise in the electronics following the photodetector. In this case, the noise can usually be modeled to high accuracy as additive, white Gaussian noise that is statistically independent of the desired signal. Let T denote the bit interval of the OOK system and assume that the receiver integrates the received photocurrent for an interval $T_0 \le T$ during each bit interval. At the end of the integration interval, the resulting electrical signal can be expressed as:

$$r_e = \eta(I_s + I_b) + n \quad (24)$$

where I_s is the received signal light intensity and I_b is the ambient light intensity. Both of these quantities can be assumed to be constant during the integration time. The optical-to-electrical conversion coefficient is given by

$$\eta = \gamma T_0 \cdot \frac{e\lambda}{hc} \quad (25)$$

where γ is the quantum efficiency of the photodetector, e is the electron charge, λ is the signal wavelength, h is Plank's constant, c is the speed of light. The additive noise n is white and Gaussian and has zero mean and covariance $N/2$, independent of whether the received bit is Off or On.

In this section, we will describe symbol-by-symbol ML detection and compute its error probability in the absence of error-correction coding. We will then describe the MLSD.

A. Symbol-by-Symbol ML Detection

1) Description: We assume that the receiver has knowledge of the marginal distribution of the turbulence-induced fading, but has no knowledge of the channel's instantaneous fading. After subtraction of the ambient light bias ηI_b, the signal $r = r_e - \eta I_b$ is described by the following conditional densities when the transmitted bit is Off or On, respectively

$$P(r|\text{Off}) = \frac{1}{\sqrt{\pi N}} \exp\left(-\frac{r^2}{N}\right) \quad (26)$$

$$P(r|\text{On}) = \int_{-\infty}^{\infty} P(r|\text{On}, X) f_X(X)dX = \int_{-\infty}^{\infty} \frac{1}{\sqrt{\pi N}} f_X(X) \cdot \exp\left[-\frac{\left(r - \eta I_0 e^{2X-2E[X]}\right)^2}{N}\right] dX. \quad (27)$$

$$C_X^T = \begin{bmatrix} \sigma_X^2 & \sigma_X^2 b_X\left(\frac{T}{\tau_0}d_0\right) & \cdots & \sigma_X^2 b_X\left[\frac{(n-1)T}{\tau_0}d_0\right] \\ \sigma_X^2 b_X\left(\frac{T}{\tau_0}d_0\right) & \sigma_X^2 & \cdots & \sigma_X^2 b_X\left[\frac{(n-2)T}{\tau_0}d_0\right] \\ \cdots & \cdots & \cdots & \cdots \\ \sigma_X^2 b_X\left[\frac{(n-1)T}{\tau_0}d_0\right] & \sigma_X^2 b_X\left[\frac{(n-2)T}{\tau_0}d_0\right] & \cdots & \sigma_X^2 \end{bmatrix}_{n\times n} \quad (21)$$

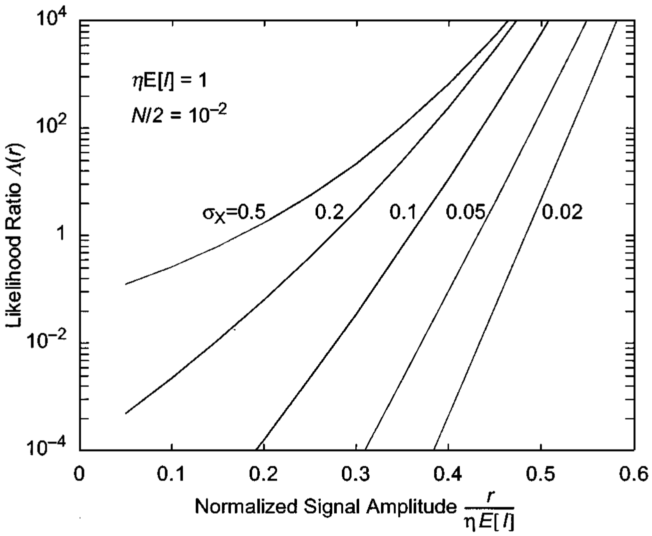


Fig. 2. Likelihood ratio versus normalized received signal amplitude for different values of the log-amplitude standard deviation.

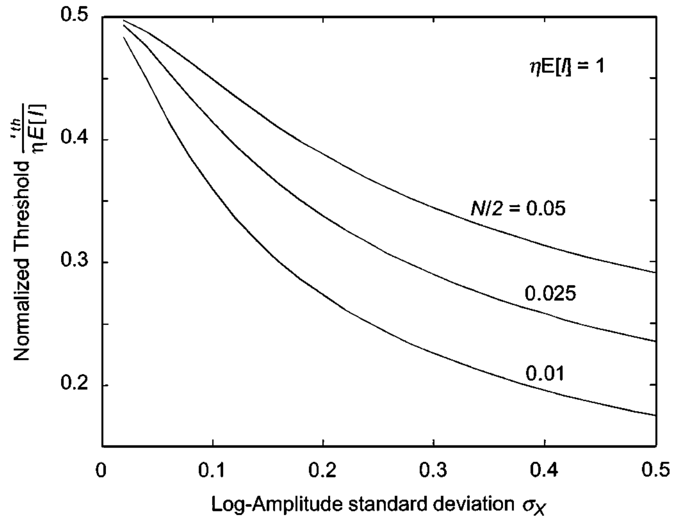


Fig. 3. Normalized threshold for maximum-likelihood symbol-by-symbol detection versus log-amplitude standard deviation for different values of the noise covariance.

The optimal maximum *a posteriori* (MAP) symbol-by-symbol detector decodes the bit $\hat{s}$ as [16], [17]

$$\hat{s} = \underset{s}{\arg\max} P(r|s)P(s) \tag{28}$$

where $P(s)$ is the probability that a On bit or Off bit is transmitted. $P(r|s)$ is the conditional distribution that if a bit s (On or Off) is transmitted, a signal level r will be received. If On and Off bits are equiprobable, or if their *a priori* probabilities are unknown, the symbol-by-symbol ML detector decodes the bit $\hat{s}$ as

$$\hat{s} = \underset{s}{\arg\max} P(r|s). \tag{29}$$

The likelihood function is

$$\begin{aligned}\Lambda(r) =& \frac{P(r|\text{On})}{P(r|\text{Off})}\\ =& \int_{\infty}^{\infty} f_X(X) \cdot \exp\left[-\frac{\left(r-\eta I_0 e^{2X-2E[X]}\right)^2 - r^2}{N}\right] dX.\end{aligned} \tag{30}$$

In Fig. 2, we see that the likelihood ratio increases monotonically with r for $0 \leq r \leq 1$, so that ML detection can be implemented by simply thresholding the received signal. In Fig. 3, we plot the optimal ML threshold for $0 \leq r \leq 1$ versus the log-amplitude standard deviation σ_X. We see that as σ_X increases, the ML threshold decreases toward zero, because turbulence-induced fading increases the fluctuation of the On-state signal level, while leaving the fluctuation of the Off-state signal level unchanged. In Fig. 3, we also see that as the additive Gaussian noise covariance $N/2$ increases, the fluctuations of the Off and On states become more closely equal and the threshold increases toward 1/2.

2) Error Probability Calculation: The bit-error probability of OOK can be computed as:

$$P_b = P(\text{Off}) \cdot P(\text{Bit Error}|\text{Off}) + P(\text{On}) \cdot P(\text{Bit Error}|\text{On}) \tag{31}$$

where $P(\text{Bit Error}|\text{Off})$ and $P(\text{Bit Error}|\text{On})$ denote the bit-error probabilities when the transmitted bit is Off and On, respectively. Without considering intersymbol interference, which can be ignored when the bit rate is not high and multipath effects are not pronounced, we have

$$P(\text{Bit Error}|\text{Off}) = \int_{\Lambda(r)>1} p(r|\text{Off})dr \tag{32}$$

$$P(\text{Bit Error})|\text{On} = \int_{\Lambda(r)<1} p(r|\text{On})dr. \tag{33}$$

B. Maximum-Likelihood Sequence Detection (MLSD)

The MLSD exploits the temporal correlation of turbulence-induced fading and is thus expected to outperform the symbol-by-symbol ML detector. For a sequence of n transmitted bits, the MLSD computes the likelihood ratio of each of the 2^n possible bit sequences $\vec{s} = [s_1\, s_2 \cdots s_n]$ and chooses

$$\begin{aligned}\vec{s} =& \underset{\vec{s}}{\arg\max} P\left(\vec{r}|\vec{s}\right)\\ =& \underset{\vec{s}}{\arg\max} \int_{\vec{X}} f_{\vec{X}}\left(\vec{X}\right)\\ & \cdot \exp\left[-\sum_{i=1}^{n} \frac{\left(r_i - \eta s_i I_0 e^{2X_i - 2E[X_i]}\right)^2}{N_i}\right] d\vec{X}.\end{aligned} \tag{34}$$

Here, each s_i can take the value Off or On, so that $s \in \{0, 1\}$.

$$\begin{aligned}f_{\vec{X}}\left(\vec{X}\right) =& \frac{1}{(2\pi)^{n/2}\left|C_X^T\right|^{1/2}}\\ & \cdot \exp\left\{-\frac{1}{2}\left[\left(X_1 - E\left[X_1\right]\right) \quad \cdots \quad \left(X_n - E\left[X_n\right]\right)\right]\right.\\ & \left. \cdot \left(C_X^T\right)^{-1} \begin{bmatrix}\left(X_1 - E\left[X_1\right]\right)\\ \cdots\\ \left(X_n - E\left[X_1\right]\right)\end{bmatrix}\right\}.\end{aligned} \tag{35}$$

The complexity of MLSD is proportional to $n \cdot 2^n$, because it requires computing an n-dimensional integral for each of 2^n bit sequences. This complexity is excessive for most applications. In a companion paper [14], we describe two reduced-complexity implementations of the MLSD, which make use of a single-step

Markov chain model for the fading correlation in conjunction with per-survivor processing.

IV. Spatial Diversity Reception

Spatial diversity reception, which has been well-studied for application at radio and microwave frequencies, has the potential to mitigate the degradation caused by atmospheric turbulence [16]–[20]. Spatial diversity reception in free-space optical communication has been proposed and studied in [18]–[20]. Ibrahim [19] has studied the performance of spatial-diversity optical reception on turbulence channels, assuming that turbulence-induced fading is uncorrelated at each of the optical receivers. In order for this assumption to hold true, the spacing between receivers should exceed the fading correlation length in the plane of the receivers. It may be difficult to satisfy this assumption in practice, for various reasons. Available space may not permit sufficient receiver spacing. In power-limited links, which often employ well-collimated beams, the receiver spacing required for uncorrelated fading may exceed the beam diameter.

A. Maximum-Likelihood Diversity Detection on Turbulence Channels

In the case of spatial diversity reception with n receivers, the received signal is described by an n-component vector $\vec{r}$. Taking account of correlation between the receivers, (26) and (27) are modified as follows:

$$P\left(\vec{r}|\text{Off}\right) = \exp\left[-\sum_{i=1}^{n}\frac{r_i^2}{N_i}\right]\prod_{1}^{n}\frac{1}{\sqrt{\pi N_i}} \tag{36}$$

$$P\left(\vec{r}|\text{On}\right) = \int_{\vec{X}} f_{\vec{X}}\left(\vec{X}\right) \cdot \exp\left[-\sum_{i=1}^{n}\frac{\left(r_i-\eta I_0 e^{2X_i-2E[X_i]}\right)^2}{N_i}\right] \cdot \prod_{1}^{n}\frac{1}{\sqrt{\pi N_i}} d\vec{X} \tag{37}$$

where $N_i/2$ is the noise covariance of the ith receiver and

$$f_{\vec{X}}\left(\vec{X}\right) = \frac{1}{(2\pi)^{n/2}\left|C_X\right|^{1/2}} \cdot \exp\left\{-\frac{1}{2}\left[(X_1-E[X_1])\ldots(X_n-E[X_n])\right] \cdot C_X^{-1}\begin{bmatrix}(X_1-E[X_1])\\ \cdots \\ (X_n-E[X_n])\end{bmatrix}\right\}. \tag{38}$$

Here, C_X is the covariance matrix of the log-amplitudes in the n receivers, as in (19). The likelihood function is

$$\Lambda\left(\vec{r}\right) = \frac{P\left(\vec{r}\ |\text{On}\right)}{P\left(\vec{r}\ |\text{Off}\right)} = \int_{\vec{X}} f_{\vec{X}}\left(\vec{X}\right) \cdot \exp\left[-\sum_{i=1}^{n}\frac{\left(r_i-\eta I_0 e^{2X_i-2[X_i]}\right)^2-r_i^2}{N_i}\right] d\vec{X}. \tag{39}$$

The ML detector employs the decision rule $\Lambda(\vec{r}) \gtrless_{\text{Off}}^{\text{On}} 1$. Since the log-amplitude follows a joint log-normal distribution, calculation of the likelihood function in (39) involves multi-dimensional integration. We emphasize that this decision rule has been derived under the assumption that the receiving party knows the fading correlation but not the instantaneous fading state.

The bit-error probability of the ML receiver is given by

$$P_b = P(\text{Off})\cdot P(\text{Bit Error}|\text{Off}) + P(\text{On})\cdot P(\text{Bit Error}|\text{On}) \tag{40}$$

where $P(\text{Bit Error}|\text{Off})$ and $P(\text{Bit Error}|\text{On})$ denote the bit-error probabilities when the transmitted bit is Off and On, respectively. Without considering intersymbol interference, which can be ignored when the bit rate is not high and multipath effects are not pronounced, we have:

$$P(\text{Bit Error}|\text{Off}) = \int_{\Lambda(\vec{r})>1} p(\vec{r}|\text{Off})d\vec{r} \tag{41}$$

and

$$P(\text{Bit Error}|\text{On}) = \int_{\Lambda(\vec{r})<1} p(\vec{r}|\text{On})d\vec{r}. \tag{42}$$

To evaluate the optimal ML diversity detection scheme, we compare it with the conventional equal-gain combining (EGC) scheme [16]. In the EGC scheme, we assume that the receiving part has knowledge of the marginal distribution of the channel fades, in each receiver, but has no knowledge of the fading correlation or the instantaneous fading state. For each individual receiver output, we can find an optimum threshold τ_i. Then the EGC detector adds together the n receiver outputs with equal gains and compares the sum to the threshold

$$T_{th} = \sum_{i=1}^{n}\tau_i. \tag{43}$$

The error probability of EGC is

$$P_b = \int_{\vec{X}} f_{\vec{X}}\left(\vec{X}\right) P\left(\text{Bit Error}|\vec{X}\right) d\vec{X} \tag{44}$$

where

$$P\left(\text{Bit Error}|\vec{X}\right) = P(\text{Off})\cdot Q\left(\frac{T_{\text{th}}}{\sqrt{2N}}\right) + P(\text{On}) \cdot Q\left[\frac{\eta I_0\left(e^{2X_1-2E[X_1]}+e^{2X_2-2E[X_2]}\right)-T_{th}}{\sqrt{2N}}\right]. \tag{45}$$

B. Numerical Simulation for Dual Receivers

In this section, we present numerical simulations of the performance of spatial-diversity detection for the dual-receiver case, which is illustrated in Fig. 4. As described above, the ML receiver [Fig. 4(a)] has full knowledge of the turbulence-induced fading correlation matrix C_X, while the EGC receiver [Fig. 4(b)] has knowledge only of the marginal distribution of fades at receivers. We assume that $E[I_1] = E[I_2] = E[I]$ and $N_1 = N_2 = N$ and define the electrical signal-to-noise ratio $SNR = (\eta E[I])^2/N$. We have used the expressions given in the previous section to numerically compute the average bit-error probabilities.

In Fig. 5(a) we plot the simulation results, assuming $E[X] = 0$ and $\sigma_X = 0.1$, varying the normalized correlation $\rho_d =$

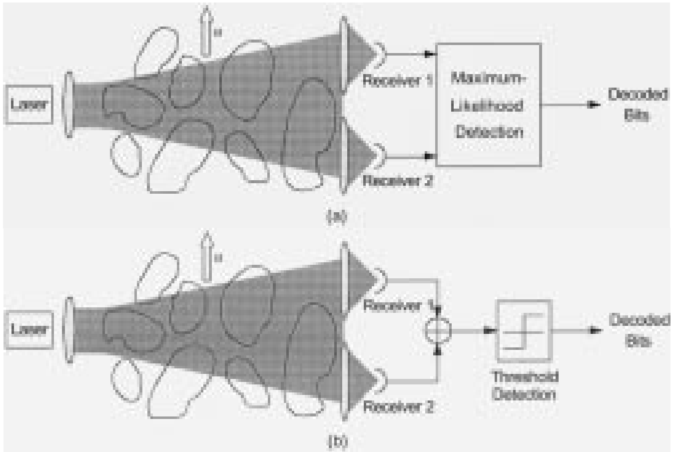
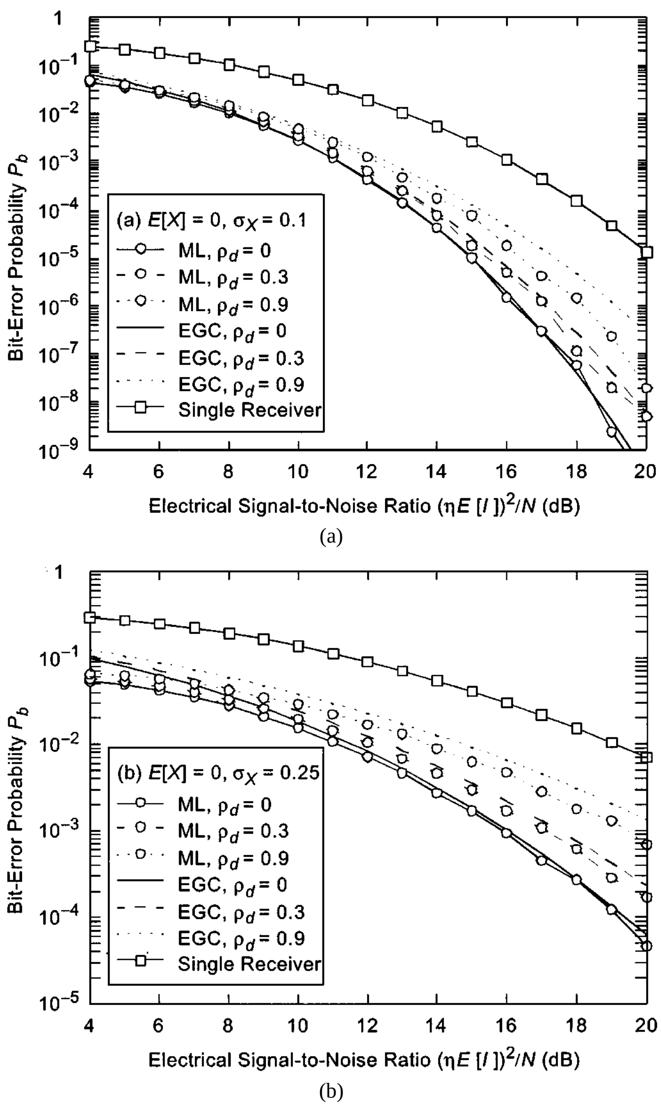


Fig. 4. Dual-branch reception on atmospheric turbulence channels with correlated turbulence-induced fading. (a) Maximum-likelihood detection. (b) Equal-gain combining with threshold detection.



(a)

(b)

Fig. 5. Bit-error probability of dual-branch receiver versus average electrical signal-to-noise ratio using maximum-likelihood detection (lines with circles) and equal-gain combining (lines without symbols) for different values of ρ_d, the normalized correlation between the two receivers. The line with squares represents the bit-error probability using a single receiver. The turbulence-induced fading has standard deviation $\sigma_X = 0.1$ in (a) and $\sigma_X = 0.25$ in (b). In both (a) and (b), this fading has mean $E[X] = 0$.

$b_X(d_{12})$ from 0 to 0.9. In Fig. 5(b), we present corresponding results assuming $\sigma_X = 0.25$. Comparing Fig. 5(a) and (b), we see that turbulence-induced fading causes a greater degradation of the bit-error probability when the standard deviation σ_X is larger. Diversity reception with two receivers can improve the performance as compared to a single receiver. With two receivers, ML detection achieves better performance than EGC for a given SNR. The advantage of ML over EGC is more pronounced when the correlation ρ_d between the two receivers is high. It is also more pronounced when the SNR is high, so that errors are caused mainly by turbulence-induced fading, as opposed to noise.

V. Conclusions

Free-space optical communication through atmosphere turbulence is now under active research and various methods have been proposed to mitigate turbulence-induced communication signal fading. In this paper we present detection techniques for free-space optical communication systems in which $D_0 < d_0$ and $T_0 < \tau_0$. These techniques can help to mitigate the effects due to turbulence-induced log-amplitude fluctuations.

The ML detection schemes over turbulence channels are studied based on the statistical distributions of turbulence-induced fading. If the instantaneous fading state is unknown but the marginal fading statistics are known, we can apply ML symbol-by-symbol detection to improve detection efficiency. If the temporal correlation of fading is known, i.e., the joint temporal distribution of turbulence-induced fading, we can apply MLSD, leading to a further improvement in detection performance.

Spatial diversity reception can also help to mitigate turbulence-induced fading. When the spacing between receivers is not much greater than the fading correlation length, diversity gain is reduced by correlation, but ML detection can be used to overcome some of this loss. We have shown that in the dual-receiver case, ML diversity reception outperforms the conventional EGC method.

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