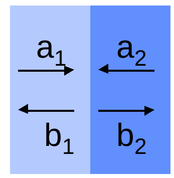
Dielectric interface

$$\mathrm{E_i}(z,t) = \mathrm{Re}\left\{a_i e^{j(wt \mp b_i z)} + b_i e^{j(wt \pm b_i z)}\right\}$$



$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} r & t \\ t & -r \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$r = \frac{n_1 - n_2}{n_1 + n_2} \qquad t = \frac{2\sqrt{n_1 n_2}}{n_1 + n_2} \qquad r^2 + t^2 = 1$$

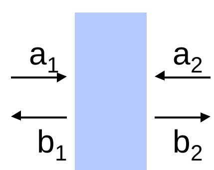
Reference plane at the interface

High - to - low: no phase shift

Lo - to - high : 180 degrees phase shift

Dielectric slab

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} r_{11} & t_{12} \\ t_{21} & r_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$



$$r_{11} = r_{22} = r_0 \frac{1 - e^{-2jq}}{1 - \left(r_0 e^{-jq}\right)^2}$$

$$t_{12} = t_{21} = e^{-jq} \frac{1 - r_0}{1 - \left(r_0 e^{-jq}\right)^2}$$

$$q = \frac{nwL}{c_0} \qquad r_0 = \frac{1-n}{1+n} \qquad |r|^2 + |t|^2 = 1$$

l/4 mirror

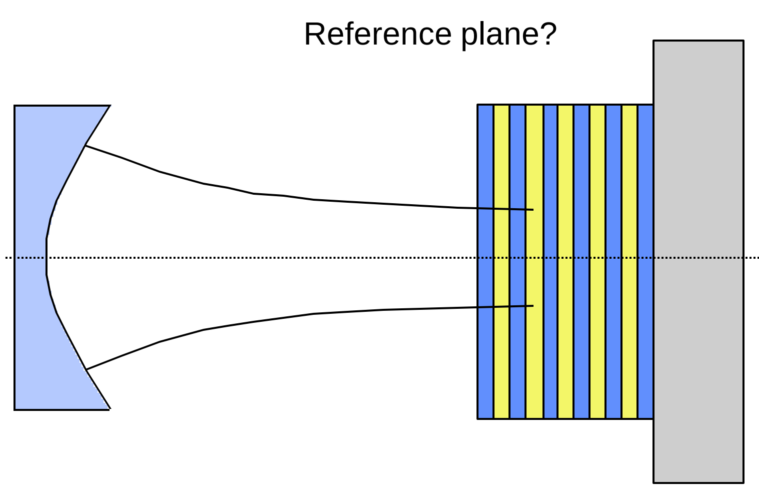
$$nL = (2m+1)\, l/4 \Rightarrow q = \frac{2pnL}{l} = (2m+1)\frac{p}{2}$$

$$e^{\pm jq} = \pm j \Rightarrow$$

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} r & jt \\ jt & r \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

Scattering matrix formalism can be extended to four ports and beyond.

Dielectric stacks



Hermitian matrices

Hermitian conjugate: $(S')_{ij} = (S)^{*}_{ji}$

Power flow:

$$P_{out} = \sum_{j=1}^{N} b^{*}_{j} b_{j} = \vec{b}'\vec{b} = (S\vec{a})'(S\vec{a}) = a'(S'S)a$$

Lossless: $S'S = I \Leftrightarrow S' = S^{-1}$

Lossless and reciprocal twoports

Lossless: $S'S = I \Leftrightarrow S' = S^{-1}$

Reciprocity: $|S_{ij}| = |S_{ji}| \Rightarrow$

$|t_{12}| = |t_{21}| \quad |r_{11}| = |r_{22}|$

$|r_{11}|^2 + |t_{21}|^2 = |r_{11}|^2 + |t_{21}|^2 = 1$

$r_{11} t^{*}_{12} + r^{*}_{22} t_{12} = 0$

$$\Rightarrow S = \begin{bmatrix} r & t \\ t & -r \end{bmatrix} \qquad S = \begin{bmatrix} r & jt \\ jt & r \end{bmatrix}$$

Polarization, transverse modes, frequency

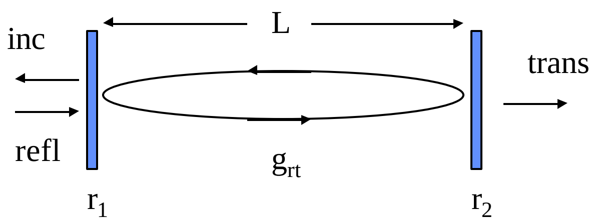
Interferometers/resonators

- Traditional optics: Fabry-Perots as optical filters
  - lateral >> longitudinal
  - plane mirrors
  - plane waves of varying incident angle
- Spatially coherent sources <-> optical resonators
  - curved mirrors reduce diffraction losses
  - transversal modes (Hermite-Gaussian or Laguerre-Gaussian)
- Unstable resonators
  - large cross sections, large diffraction loss
- Passive cavities

Resonator modes

- Standing wave cavities
- Ring cavities
  - one direction -> no standing waves
  - less reflection into master oscillator
- Mode matching
  - ~ TEM
  - overlap integral
- Uniform plane wave approximation

Circulating intensity - plane wave approximation



$E_{circ} = jt_1 E_{inc} + g_{rt} E_{circ}$

$g_{rt} = r_1 r_2 r_3 \ldots \exp(-\alpha_0 p - j\omega p/c)$

Cavity resonance

$$\frac{E_{circ}}{E_{inc}} = \frac{jt_1}{1 - g_{rt}(\omega)} = \frac{jt_1}{1 - r_1 r_2 r_3 \ldots e^{-\alpha_0 p - j\omega p/c}}$$

Axial modes, free spectral range

$$\left.\frac{E_{circ}}{E_{inc}}\right|_{\omega=\omega_q} = \frac{jt_1}{1 - r_1 r_2 e^{-\alpha_0 p}} \approx \frac{jt}{1 - r^2} = \frac{j}{t} \Rightarrow \left.\frac{I_{circ}}{I_{inc}}\right|_{\omega=\omega_q} \approx \frac{1}{T}$$

$$\left.\frac{I_{circ}}{I_{inc}}\right|_{\omega=\omega_q} = \frac{t_1 t_2}{\left(1 - r_1 r_2 e^{-\alpha_0 p}\right)^2}$$

$$\frac{E_{trans}}{E_{inc}} = \frac{-t_1 t_2 e^{-\alpha_0 p_1 - j\omega p_1/c}}{1 - r_1 r_2 r_3 \ldots e^{-\alpha_0 p - j\omega p/c}}$$

Reflections

$$E_{refl} = r_1 E_{inc} + jt_1 \frac{g_{rt}}{r_1} E_{circ} \Rightarrow$$

$$\frac{E_{refl}}{E_{inc}} = r_1 - \left[\frac{jt_1^2 r_2}{1 - r_1 r_2 r_3 \ldots . e^{-a_0 p - jwp/c}} \right] = r_1 - \frac{t_1^2}{r_1} \frac{g_{rt}(w)}{1 - g_{rt}(w)}$$

$$= \frac{1}{r_1} \frac{r_1^2 - g_{rt}(w)}{1 - g_{rt}(w)} = \frac{r_1 - \frac{g_{rt}(w)}{r_1}}{1 - r_1 \frac{g_{rt}(w)}{r_1}}$$

$$\frac{g_{rt}(w)}{r_1} = e^{jj} \Rightarrow \frac{E_{refl}}{E_{inc}} = \frac{r_1 - e^{jj}}{1 - r_1 e^{jj}} = -e^{jj} \frac{1 - r_1 e^{-jj}}{1 - r_1 e^{jj}}$$

Summary

- Mirrors and dielectric slabs
- Hermitian matrices
- Fabry-Perots
- Resonators and Cavity resonance
- Circulating, transmitted and reflected field and intensity from optical resonators
- Trial midterm