Pilot-Symbol Assisted Modulation for Correlated Turbulent Free-Space Optical Channels

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Abstract

In free-space optical links using intensity modulation and direct detection, atmospheric turbulence can cause signal fading. Pilot-symbol (PS) assisted modulation (PSAM) can help to mitigate this fading, improving system performance. We periodically insert an On-state PS in front of M-1 information bits and form a M-bit frame. We derive the PS assisted maximum-likelihood (PSA-ML) decision rule under the assumption that the temporal coherence of fading is known, but the instantaneous fading state is not known. We also propose a simpler PS assisted detection scheme with variable threshold (PSA-VT). Although these two techniques will introduce some delay to the detection system, they can help to mitigate the atmospheric turbulence induced fading. We have performed numerical simulations to show this improvement. We also describe how to choose the frame size M to optimize the performance of PSAM systems.

Keywords: Free-space optical communication, atmospheric turbulence, pilot-symbol assisted modulation (PSAM)

1. Introduction

Free-space optical communication can provide high-speed line-of-sight communication links in many applications [1]-[4]. However, atmospheric turbulence will greatly deteriorate the received signal, limiting the performance of the communication system [5]-[7]. The temporal and spatial atmospheric turbulence-induced channel fading has been studied to help the detection of free-space optical communication system through correlated turbulence [5]. Pilot-symbol assisted modulation (PSAM) has been employed in applications at radio and microwave frequencies to mitigate the effects of channel fading in situations where channel side information (SI), i.e., the instantaneous fading state, is not known to the receiver [8][9]. In PSAM, the transmitter periodically inserts known symbols, which contains reference for receiver to derive the correlated channel fading. PSAM can also be incorporated with maximum likelihood sequence detection (MLSD) to truncate the survivor path of information bit string [10].

In this paper we first introduce the system model of IM/DD free-space optical communication links through atmospheric turbulence. Then we will propose the pilot-symbol assisted maximum likelihood (PSA-ML) detection as well as another simpler PS assisted detection scheme with variable threshold (PSA-VT). We present the numerical simulations for these detection techniques and compare them with the normal ML symbol-by-symbol detection.

2. System Model

Because of the complexity associated with phase or frequency modulation, current free-space optical communication systems typically use intensity modulation with direct detection (IM/DD). In this paper, we consider IM/DD links using On-Off Keying (OOK). In most practical systems, the receiver signal-to-noise ratio (SNR) is limited by shot noise caused by ambient light which is much stronger than the desired signal and/or by thermal noise in the electronics following the photodetector. In this case, the noise can usually be modeled to high accuracy as additive, white Gaussian noise that is statistically independent of the desired signal. Assume that the receiver integrates the received photocurrent for an interval $T_0 \le T$ during each bit interval and $T_0 \ll \tau_0$, where τ_0 denotes the coherence time for atmospheric turbulence. Therefore the light intensity can be viewed constant during each exposure interval. At the end of the integration interval, the resulting electrical signal can be expressed as:

$$r_e = \eta(I_s + I_b) + n, \tag{1}$$

where I_s is the received signal light intensity and I_b is the ambient light intensity. The optical-to-electrical conversion efficiency is given by

$$\eta = \gamma T_0 \cdot \frac{e\lambda}{hc}, \tag{2}$$

where γ is the quantum efficiency of the photodetector, e is the electron charge, λ is the signal wavelength, h is Plank's constant, and c is the speed of light. The additive noise n is white and Gaussian, and has zero mean and covariance $N/2$, independent of whether the received bit is Off or On.

Ignoring intersymbol interference (ISI), the receiver would only receive signal light through turbulence when On-state is transmitted. The i th On-state symbol intensity can be expressed as:

$$I_i = I_0 \exp(2X_i - 2\chi), \tag{3}$$

where X_i is so-called log-amplitude of the optical signal and can be modeled as a Gaussian random variable with a mean of $\chi = E[X_i]$ and covariance σ_X^2. The temporal joint probability distribution function (PDF) of log-amplitude sequence $\vec{X} = [X_{n_1} - \chi, X_{n_2} - \chi, \ldots, X_{n_m} - \chi]$ is jointly Gaussian [5]:

$$f_{\vec{X}}(\vec{X}) = \frac{1}{(2\pi)^{m/2}\left|C_X^{\mathrm{On}}\right|^{1/2}} \exp\left[-\frac{1}{2}\vec{X} \cdot (C_X^{\mathrm{On}})^{-1} \cdot \vec{X}^T\right], \tag{4}$$

where C_X^{On} is the covariance matrix of On-state bits sequence:

$$C_X^{\mathrm{On}} = \begin{bmatrix} \sigma_X^2 & \sigma_X^2 \exp\left[-\left(\frac{|n_1 - n_2|T}{\tau_0}\right)^{5/3}\right] & \ldots & \sigma_X^2 \exp\left[-\left(\frac{|n_1 - n_m|T}{\tau_0}\right)^{5/3}\right] \\ \sigma_X^2 \exp\left[-\left(\frac{|n_2 - n_1|T}{\tau_0}\right)^{5/3}\right] & \sigma_X^2 & \ldots & \sigma_X^2 \exp\left[-\left(\frac{|n_2 - n_m|T}{\tau_0}\right)^{5/3}\right] \\ \ldots & \ldots & \ldots & \ldots \\ \sigma_X^2 \exp\left[-\left(\frac{|n_m - n_1|T}{\tau_0}\right)^{5/3}\right] & \sigma_X^2 \exp\left[-\left(\frac{|n_m - n_2|T}{\tau_0}\right)^{5/3}\right] & \ldots & \sigma_X^2 \end{bmatrix}_{n \times n}, \tag{5}$$

where T is the bit interval. The coherence time is

$$\tau_0 = d_0 / u_\perp, \tag{6}$$

where d_0 is the coherence diameter of turbulence-induced fading and $u_\perp$ is the wind velocity perpendicular to the light propagation direction. Here we assume u to be constant and ignore its fluctuations.

The joint distribution of the signal intensity of m On-state symbols is therefore joint log-normal [5]:

$$f_{\vec{I}}(I_{n_1}, I_{n_2}, \ldots, I_{n_m}) = \frac{1}{2^m \prod_{i=1}^{m} I_{n_i}} \frac{1}{(2\pi)^{m/2}\left|C_X^{\mathrm{On}}\right|^{1/2}} \exp\left\{-\frac{1}{8}\left[\ln\left(\frac{I_{n_1}}{I_0}\right) \;\ldots\; \ln\left(\frac{I_{n_m}}{I_0}\right)\right] (C_X^{\mathrm{On}})^{-1} \begin{bmatrix} \ln\left(\frac{I_{n_1}}{I_0}\right) \\ \ldots \\ \ln\left(\frac{I_{n_m}}{I_0}\right) \end{bmatrix}\right\}. \tag{7}$$

3. Pilot-Symbol Assisted Maximum-Likelihood Detection

In this section, the normal ML symbol-by-symbol detection scheme is introduced first. The PSA-ML and PSA-VT detection techniques are proposed afterwards by periodically inserting On-state symbols to the transmitted information OOK bit-sequence.

A. Maximum-likelihood symbol-by-symbol detection

The optimum *maximum a posteriori* (MAP) receiver decodes the symbol $\hat{s}$ as:

$$\hat{s} = \underset{s}{\arg\max}\, p(r|s)P(s), \tag{8}$$

where $p(r|s)$ is the density of the reception when symbol s is transmitted, and $P(s)$ is the prior probability of symbol s. If the prior probabilities of all possible symbols are equal or are not known, MAP detection reduces to ML, which decodes the symbol $\hat{s}$ as:

$$\hat{s} = \underset{s}{\arg\max}\, p(r|s). \tag{9}$$

In what follows, we assume the modulation is OOK, so that $s \in \{0, 1\}$.

B. Pilot-symbol assisted maximum-likelihood detection

For PSAM, we periodically insert On-state symbols to the information bit string. The composite symbols are transmitted in the usual way over the channel. The resulting frame structure is shown in Fig. 1. At the receiver, we can decode the information bit by considering the joint distribution of the turbulence induced fading at the information bit and the adjacent pilot symbols. Although PSAM will cause delay in the receiver for storing the whole frame before decoding, the pilot-symbols (PS) provide receiver with explicit atmospheric turbulence induced fading reference for detection and can help to mitigate the effects of fading.

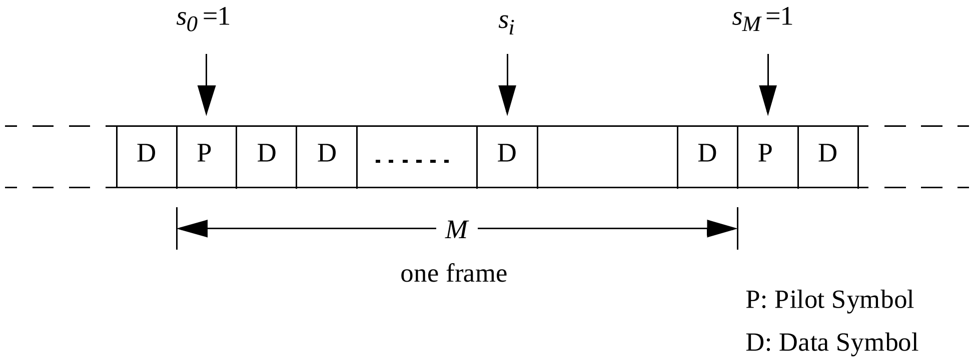

Fig. 1. Pilot-symbol assisted modulation frame structure.

In Fig. 1, the composite frame is of length *M*, the first symbol of each frame is the On-state PS followed by *M*-1 information symbols. When we try to detect the information bits in one frame, we could refer to the PS of current frame and the PS of the next frame. Assume r_i is the received photo-current signal of *i*th information bit in the frame, r_0 and r_M are the received signal of the PS of the current frame and the next frame. The joint probability distribution of $\vec{r} = [r_0, r_i, r_M]$ conditioned on the *i*th $(1 \le i \le M-1)$ information bit s_i are:

$$p(\vec{r}|s_i = 0) = \frac{1}{(\pi N)^{3/2}} \exp\left[-\frac{r_i^2}{N}\right] \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{\vec{X}}(X_0, X_M) \exp\left[-\sum_{j=0,M} \frac{(r_j - \eta I_0 e^{2X_j - 2\chi})^2}{N}\right] dX_0 dX_M, \tag{10}$$

$$p(\vec{r}|s_i = 1) = \frac{1}{(\pi N)^{3/2}} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{\vec{X}}(X_0, X_i, X_M) \exp\left[-\sum_{j=0,i,M} \frac{(r_j - \eta I_0 e^{2X_j - 2\chi})^2}{N}\right] dX_0 dX_i dX_M. \tag{11}$$

Where the PDF of log-amplitude $f(X_0, X_M)$ and $f(X_0, X_i, X_M)$ are shown as in (4)-(5).

The likelihood ratio is:

$$\Lambda(\vec{r}) = \frac{p(\vec{r}|s_i = 1)}{p(\vec{r}|s_i = 0)} = \frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{\vec{X}}(X_0, X_i, X_M) \exp\left[-\sum_{j=0,i,M} \frac{(r_j - \eta I_0 e^{2X_j - 2\chi})^2}{N}\right] dX_0 dX_i dX_M}{\exp\left[-\frac{r_i^2}{N}\right] \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{\vec{X}}(X_0, X_M) \exp\left[-\sum_{j=0,M} \frac{(r_j - \eta I_0 e^{2X_j - 2\chi})^2}{N}\right] dX_0 dX_M}. \tag{12}$$

The ML detector employs the decision rule $\Lambda(\vec{r}) \underset{0}{\overset{1}{\gtrless}} 1$. We emphasize that this decision rule has been derived under the assumption that the receiver knows the fading correlation but not the instantaneous fading state.

The bit-error probability of *i*th information bit in *M*-bit frame is given by:

$$P_i = P(s_i = 0) \cdot P(\text{Bit Error}|s_i = 0) + P(s_i = 1) \cdot P(\text{Bit Error}|s_i = 1)\,, \tag{13}$$

where $P(\text{Bit Error}|s_i = 0)$ and $P(\text{Bit Error}|s_i = 1)$ denote the bit-error probabilities when the transmitted *i*th information bit is 0 and 1, respectively. Without considering intersymbol interference, which can be ignored when the bit rate is not high and multipath effects are not pronounced, we have:

$$P(\text{Bit Error}|s_i = 0) = \int_{\Lambda(\vec{r}) > 1} p(\vec{r}|s_i = 0)d\vec{r} \tag{14}$$

and

$$P(\text{Bit Error}|s_i = 1) = \int_{\Lambda(\vec{r}) < 1} p(\vec{r}|s_i = 1)d\vec{r}\,. \tag{15}$$

The average bit-error probability of one frame is:

$$P_{\text{b}} = \frac{1}{M-1}\sum_{i=1}^{M-1} P_i\,. \tag{16}$$

C. Pilot-symbol assisted symbol-by-symbol detection with variable threshold

PSA-ML detection described above employs the joint PDF of turbulence induced fading however it needs very lengthy computations to do multiple-dimensional integration. Here we propose a simpler PS assisted detection scheme with variable threshold set by received pilot-symbols (PSA-VT).

Since $[X_0, X_i, X_M]$ is of joint Gaussian distribution, $E[X_i|X_0, X_M]$ is an affine function of the log-amplitude at two adjacent PS positions: X_0 and X_M:

$$E[X_i|X_0, X_M] = a_i^0 X_0 + a_i^M X_M\,, \tag{17}$$

$$\begin{bmatrix} a_i^0 \\ a_i^M \end{bmatrix} = \begin{bmatrix} 1 & \exp\left[-\left(\frac{mT}{\tau_0}\right)^{5/3}\right] \\ \exp\left[-\left(\frac{mT}{\tau_0}\right)^{5/3}\right] & 1 \end{bmatrix}^{-1} \begin{bmatrix} \exp\left[-\left(\frac{iT}{\tau_0}\right)^{5/3}\right] \\ \exp\left[-\left(\frac{(M-i)T}{\tau_0}\right)^{5/3}\right] \end{bmatrix}. \tag{18}$$

We simply assume the received PS signals are noise free. Therefore we can set the variable threshold for *i*th $(1 \le i \le M-1)$ information bit in the frame to be:

$$\tau_i = \frac{(r_0)^{a_i^0}(r_M)^{a_i^M}}{2}\,. \tag{19}$$

Using PSA-VT, the bit-error probability of the *i*th information bit in a *M*-bit frame is given by:

$$P_i = P(s_i = 0) \cdot P(\text{Bit Error}|s_i = 0) + P(s_i = 1) \cdot P(\text{Bit Error}|s_i = 1)\,, \tag{20}$$

$$P(\text{Bit Error}|s_i = 0) = \int_{\vec{r},\, r_i > \tau_i} p(\vec{r}|s_i = 0)d\vec{r}\,, \tag{21}$$

$$P(\text{Bit Error}|s_i = 1) = \int_{\vec{r},\, r_i < \tau_i} p(\vec{r}|s_i = 1)d\vec{r}\,. \tag{22}$$

The expression for the average bit-error probability is the same as (16).

4. Numerical Simulation

In this section, we present numerical simulations of the detection schemes we proposed in Section 3. Assume the turbulence induced log-amplitude fluctuation has zero mean, covariance $\sigma_X = 0.1$ and $T/\tau_0 = 0.001$. If the receiver has knowledge only of the marginal distribution of fades but not the instantaneous fading at the receiver, the bit-error probability of ML symbol-by-symbol detection versus the average electrical signal-to-noise ratio $SNR = (\eta I_0)^2/N$ is shown in Fig. 2 with dot line. If the SI of instantaneous channel fading is known to the receiver, the symbol-by-symbol ML detection could achieve better bit-error performance and the corresponding simulation result is plotted in Fig. 2 with dash-dot line. However such SI of

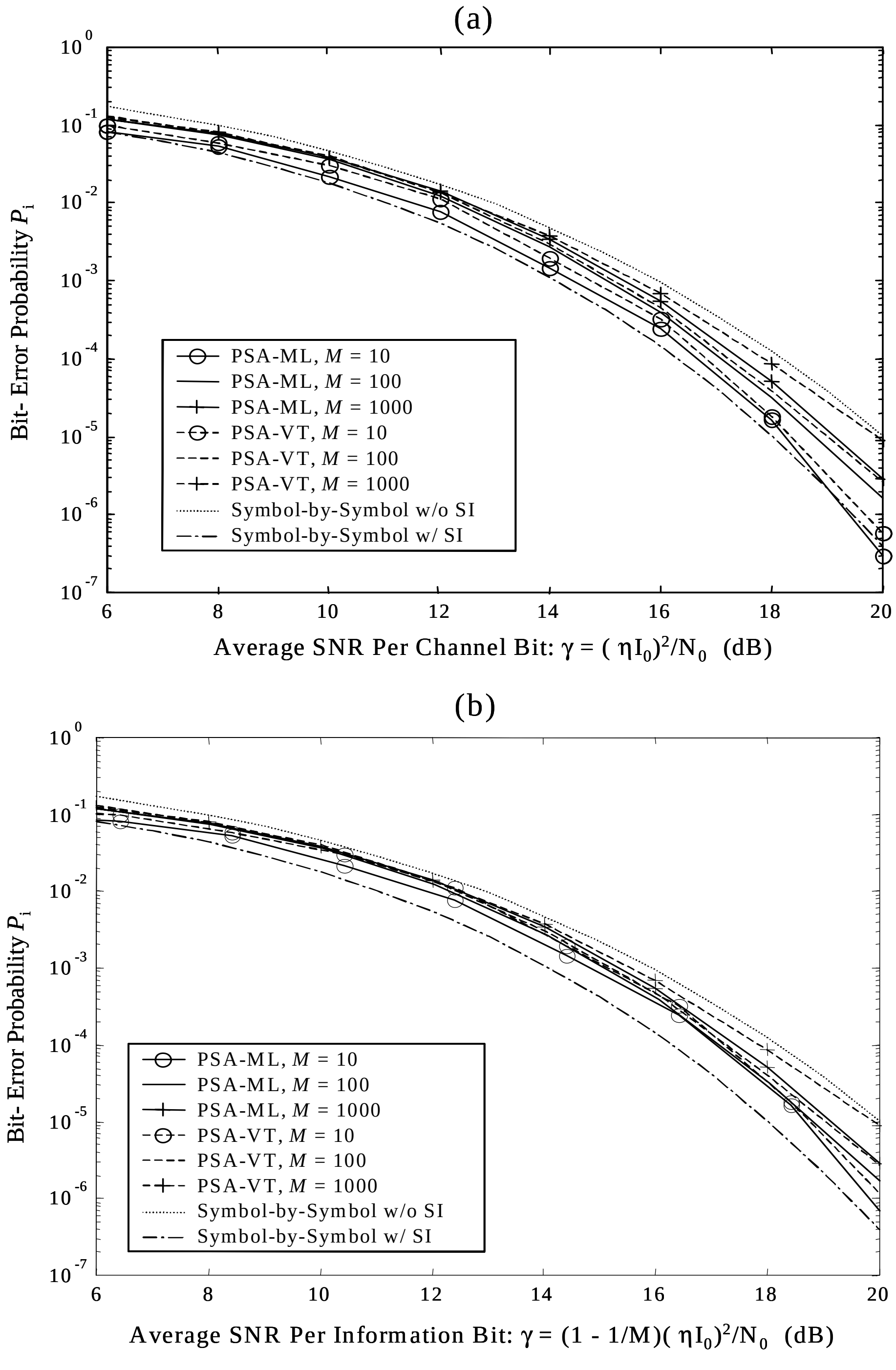

Fig. 2. Bit-error probability of *i*th symbol in a *M*-bit frame versus (a) average electrical SNR of each channel bit and (b) average electrical SNR of each information bit using PSA-ML and PSA variable threshold (VT). We choose different values of frame size *M* (10, 100 and 1000 respectively). Here $i = M/2$. The covariance of log-amplitude due to turbulence-induced fading is $\sigma_X = 0.1$. The ratio of bit-interval *T* versus fading coherence time τ_0 is $T/\tau_0 = 0.001$. The BER of ML symbol-by-symbol detection with or without instantaneous channel fading side-information (SI) are also shown in the figure for comparison.

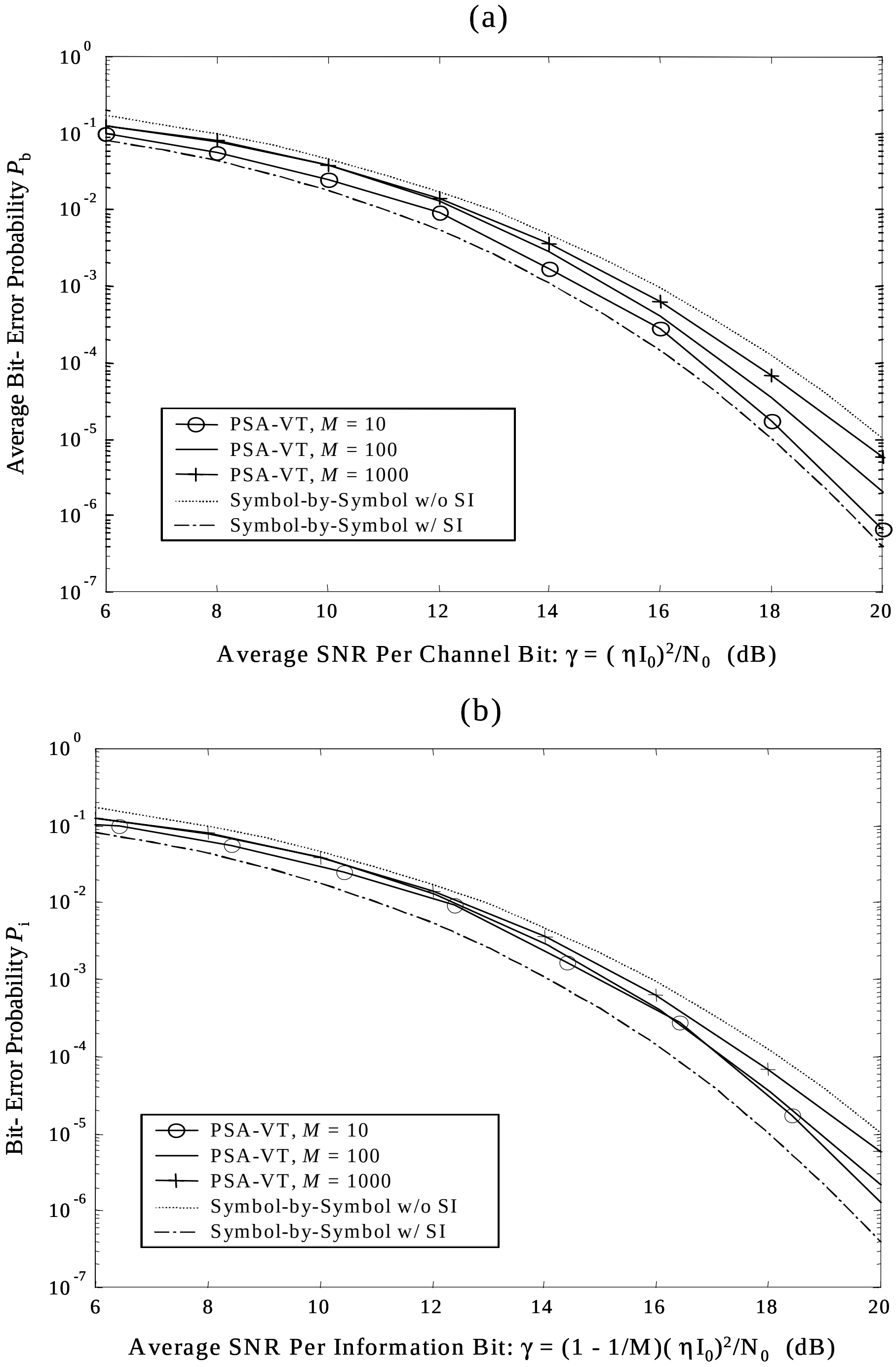

Fig. 3. Average bit-error probability of a M-bit frame versus (a) average electrical SNR of each channel bit and (b) average electrical SNR of each information bit using PSA-VT detection. We choose the frame size *M* to be 10, 100 and 1000, respectively. The covariance of log-amplitude fluctuation due to turbulence-induced fading is $\sigma_X = 0.1$. Here $i = M/2$. The covariance of log-amplitude due to turbulence-induced fading is $\sigma_X = 0.1$. The ratio of bit-interval T versus fading coherence time τ_0 is $T/\tau_0 = 0.001$. The BER of ML symbol-by-symbol detection with or without instantaneous channel fading side-information (SI) are also shown in the figure for comparison.

channel fading state is often not known to the receivers. If we assume the receiver has full knowledge of the joint probability distribution for temporal correlated turbulence-induced fading, PS assisted detection schemes discussed in Section 3-B and C could be applied. In our simulation, we use the PSAM frame structure shown in Fig. 1. In Fig. 2, we plot the bit-error probability of the *i*th ($i = M/2$) symbol in a *M*-bit frame with PSA-ML detection and PS assisted variable threshold (PSA-VT) detection respectively. In Fig. 2-(a), we plot the BER versus the average electrical SNR of each channel bit and in Fig. 2-(b), we plot the BER versus the average electrical SNR of each information bit. We assume the frame size *M* to be 10, 100 and 1000 respectively. In Fig. 3-(a) and (b), we plot the average bit-error probability of a *M*-bit frame with PSA-VT detection versus the average electrical SNR of each channel bit and of each information bit respectively, varying the frame size *M* to be 10, 100 and 1000.

From Fig. 2 and Fig. 3, we observe that when the receiver knows the temporal coherence of channel fading however not the SI of instantaneous channel fading, PS assisted detection techniques can give better bit-error performance than ML symbol-by-symbol detection. However with larger frame size, the coherence between the PS and information symbols will become weaker. This will make the PS assisted detection less effective. In our simulation, we observe that to ensure PS assisted detection achieve a gain of at least 0.5 dB over ML symbol-by-symbol detection, we need $M < \tau_0 / T$. In peak power limited systems, assume the channel bit-rate is the same, smaller *M* will help the BER performance as shown in Fig. 2-(a) and Fig. 3-(a). Another benefit of choosing smaller *M* is that it will cause less delay during detection. However to transmit additional PS, we will waste some bandwidth.

On the other hand, for average power limited systems, inserted PS will also consume additional power and decrease the SNR of each channel-bit. As we can see in Fig. 2-(b) and Fig. 3-(b), the BER performance versus the average SNR per information bit for *M* = 10 and *M* = 100 is very close. If we choose *M* even smaller, the penalty due to the additional power to transmit PS will surpass the PSAM gain. Therefore in average power limited systems, we can not choose *M* to be too small.

In Fig. 2, we also note PSA-ML can achieve better bit-error performance than PSA-VT. However PSA-VT does not need multi-dimensional integration and is much faster and simpler to implement than PSA-ML detection rule.

5. Conclusions

We have studied the problem of IM/DD free-space optical communication in the presence of turbulence-induced fading. Pilot-symbol assisted modulation (PSAM) can mitigate this fading, improving system performance. We propose the PSA-ML and PSA-VT detection schemes under the assumption that the fading correlation properties are known, but the instantaneous fading state is not known. We have performed numerical simulations for both techniques. Our results show that both PS assisted detection can provide better bit-error performance than the ML symbol-by-symbol detection to mitigate correlated channel fading.

6. Acknowledgment

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7. References

1. V. Hsu, J. M. Kahn, and K. S. J. Pister, “Wireless Communications for Smart Dust”, Electronics Research Laboratory Technical Memorandum Number M98/2, February, 1998.
2. J. M. Kahn, R. H. Katz and K. S. J. Pister, “Mobile Networking for Smart Dust”, *Proc. of ACM/IEEE Intl. Conf. on Mobile Computing and Networking* (MobiCom 99), Seattle, WA, August 17-19, 1999.
3. T. H. Carbonneau, D. R. Wisely, “Opportunities and challenges for optical wireless; the competitive advantage of free-space telecommunications links in today’s crowded marketplace”, *Wireless Technologies and Systems: Millimeter-Wave and Optical, Proc. SPIE,* Vol. 3232, 1997, pp. 119-128.
4. A. Annamalai, C. Tellambura, V. K. Bhargava, “A simple and accurate analysis of digital communication systems with diversity reception in different fading environments”, *Proc. of Ninth International Symp. on Personal, Indoor, and Mobile Radio Commun.* (PIMRC’98), New York, NY, USA; vol.3.3, vol. 1574, p.1055-60, Sep. 1998.
5. X. Zhu, J. M. Kahn, “Free-Space Optical Communication with Imaging Receivers through Atmospheric Turbulence Channels”, subm. to *IEEE Trans. on Commun.*, August 2000.
6. Joseph W. Goodman, *Statistical Optics*, John Wiley & Sons, Inc., 1985.
7. S. Karp, R. Gagliardi S. E. Moran and L. B. Stotts, *Optical Channels*, Plenum Press, 1988.
8. J. K. Cavers, “An Analysis of Pilot Symbol Assisted Modulation for Rayleigh Fading Channels”, *IEEE Trans. on Commun.*, Vol 40, No. 4, pp. 686-93, Nov. 1991.
9. S. B. Bulumulla, S. A. Kassam, S. S. Venkatesh, “Pilot symbol assisted diversity reception for a fading channel”, *Proceedings of the 1998 IEEE International Conference on Acoustics, Speech and Signal Processing*, Seattle, WA, USA, Vol. 6, p.3417-20, May 1998.
10. X. Zhu and J.M. Kahn, “Markov Chain Model in Maximum-Likelihood Sequence Detection for Free-Space Optical Communication through Atmospheric Turbulence Channels”, subm. to *IEEE Trans. on Commun.*, August 2000.